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Arithmetic in the Total School Program

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EFFECTIVE ARITHMETIC INSTRUCTION is not confined solely to arithmetic periods. The demands placed upon the arithmetical knowledge of the children are inherent in all subjects in the elementary school curriculum. In some areas of study these demands are enormous while in others they are less significant. However, if we can generalize at all, we can say that knowledge of arithmetic or a knowledge of quantity is necessary in every subject that is taught in the elementary school. Indeed, we might go further to say that the understandings derived from studying the sciences, social studies, even spelling, music and art are dependent in some measure upon the quantity involved.

Brownell cites three large areas that might comprise desirable arithmetic outcomes.[1] They are: (1) computational skill; (2) mathematical understanding; (3) sensitiveness to number in social situations and a habit of using number effectively in such situations. Should these outcomes be assiduously sought in arithmetic programs, it would be no exaggeration to state that arithmetic instruction would have come of age. A program characterized by such objectives would be a balanced one, and the balance would be struck by the application of arithmetic understandings in the other areas of the curriculum.

[1] Brownell, W. A., "When Is Arithmetic Meaningful?" *Journal of Educational Research*, March, 1945, 481–498.

Curriculum workers commonly define the curriculum as those experiences offered under the supervision of the school. It would then be fair to assume that the arithmetic curriculum would be those quantitative or number situations that are encountered anywhere in the school day. Thus, there would be a time and a place for systematic teaching of arithmetic and the appropriate knowledge applied when demanded to further understanding in other curricular fields. Perhaps this observation might be in order . . .

> "An arithmetic course which does not have as a very definite objective the development of ability to compute with accuracy and facility is wholly inadequate. However, the curriculum which tries to accomplish nothing but accuracy and facility in figuring is woefully one-sided. Moreover, a curriculum which sets out to do that and only that will not even serve its own purpose satisfactorily."[2]

The Place of Arithmetic in the Curriculum

The place of arithmetic in the curriculum is conditioned somewhat by the curriculum pattern in vogue in various school systems. If a school system is dedicated to an integrated program, the program in arithmetic instruction will differ from that of a school which is concerned with a more subject-centered curriculum. There is a good deal

[2] Horn, Ernest, "Arithmetic in the Elementary School Curriculum" *The Teaching of Arithmetic*, 50th Yearbook, Part II, National Society for the Study of Education, University of Chicago Press, 1951.

written about experience units, child centered curricula and so forth in the literature these days. However, in examining this literature it is a bit difficult to determine just what these terms mean and, consequently, it is a bit difficult to determine what the place of subjects in these kinds of curriculum might be. One of the complicating factors lies in the fact that many experience units are advertised as being integrated when in fact, the arithmetic or any other subject in the curriculum is taught systematically and in an orderly fashion. In a subject centered curriculum we are less doubtful about the place of arithmetic in the curriculum for it has a special place and time is set aside for it. However, the point remains that the quality and kind of arithmetic instruction will be heavily conditioned by the particular type of curriculum organization in effect.

No matter what kind of curriculum organization is in vogue there are still some considerations that must be faced. For example as Horn points out, the kinds of arithmetic experiences that are to be offered in the content fields should be determined by the nature of learning undertaken in these fields. Units in science and in the social studies, if well taught, will not be able to avoid arithmetic concepts and abilities. This is true whether the curriculum is organized in terms of subjects or in terms of an integrated curriculum or experience units. It might be further pointed out that in these situations the arithmetical understandings are often crucial to the success of the learning in these fields. In dealing with this problem, Horn has suggested that such questions as these need to be raised:

1. What are the nature and amount of the demands which other areas make upon arithmetic?
2. How adequate are the pupils' abilities to meet these demands?
3. What are the probable or potential contributions of these areas to the development of arithmetical abilities?
4. Is there a need for specially planned instruction in arithmetic and, if so, what is the relation of that instruction to the arithmetical instruction incident to the attack on the mathematical aspects of other areas?[3]

Frequency of Arithmetical Terms in Texts and References in Other Fields in the Elementary School Curriculum

Practically all investigations concerned with this particular problem have indicated that the number of arithmetical terms in other elementary school subjects is quite extensive. However, this extensiveness is dependent somewhat upon how the arithmetic term is defined. If marginal terms such as more, or none, or often are included, then there are very many indeed. For example, one investigation which looked into geography books indicated that one out of every seven running words was concerned with some idea of quantity.[4] The most casual perusal of social studies materials will reveal the enormous number of quantitative statements that are encountered. The following is taken from a primary social studies text:

> "Eight-year-old Sally sat in the doorway knitting. Knit, knit, knit! Was there ever an end to it?
>
> There were two brothers to knit stockings for. Sally's fingers were never still. She must work all summer to be ready for winter.
>
> In winter, feet were often cold and wet. There were no rubbers or overshoes in those days. There were no hard, dry sidewalks. In Jamestown there were only dirt paths and they were often muddy. Warm knitted stockings were a comfort to cold feet in wet shoes.
>
> Knit, knit, knit! Sally's fingers flew.
>
> 'Sally!' called her mother. 'Watch the kettle that is hanging over the fire. Don't let the dinner burn. And mind the baby. I'm taking the clothes to the river to wash.'"

In just this one item of reading there are eleven quantitative ideas, most of them highly unspecific. This comprises approximately eight per cent of these running words, and it might be normally expected, as books get a little more difficult in terms of readability, that this percentage would increase. The following passage from an upper grade text will illustrate this.

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[3] *Op. cit.*, page 12.

[4] *Ibid.*

"Not many years ago there were few symphony orchestras in the United States. Today there are many, and quite a number of them may be heard over the air in regular weekly programs. The radio has accustomed more people than ever to the music of orchestras, and the United States has probably taken first place as a nation of music lovers and music sponsors. Music has come to be a part of American life, and a large part of our recreation and entertainment is connected with music."

The number of quantitative elements in this short selection is extremely heavy. There are approximately eighty-eight words and sixteen of these eighty-eight words, or eighteen per cent, are quantitative in nature. Moreover, this is complicated by the fact that these terms do not exist in isolation. Notice the very first few words—"not many years ago"—a little further on we come to—"quite a number." These are ideas that are not specific and consequently difficult to appraise. It is, perhaps, unfortunate that many text books deal with these unspecific terms rather than dealing in precise statements of quantity. Then children could come to grips with them and show either manipulatively or by our number system just what is meant by statements such as these. There is evidence that indicates that children have difficulty with this kind of quantitative statement. For example, some years ago in a study conducted by Buswell and John[5] it it was pointed out that children's understanding of quantitative terms was weak. For example, the percentages of correct responses in grades four to six for the following terms were these: Acre—38.9%; area—28%; average—43%, difference—71%; rectangle—33%. More pupils thought an acre to be larger than a square mile than believed it to be smaller than a city block, and among the most flagrant misconceptions might be these: Acorns—an acre is acrons; 100,000 miles is an acre; acre is a man's name; houses—acres are houses, and one child, working on the principle of assonance, decided that an acre was to have a stomach ache. The terms tested in this particular study were taken from elementary school text books and, as a result of this study, the authors recommend that increased attention be given to the development of common arithmetical terms. Another study[6] (Horn's Methods of Instruction in the Social Studies) that looked into definite and indefinite terms came up with these examples: Ten square miles meant to the various pupils—about the size of Chicago; about the size of the state of Iowa; about the size of Washington Park; about as large as ten acres; about one lot; and one child said without any idea of square measure, "here to Key West in a straight line." Getting back to indefinite terms such as a great deal; many; far; thick; it might be shown that these are more difficult than are definite statements of quantity. A few years ago the writer became interested in this problem and devised a test for a group of sixth grade children of superior ability. These children had completed a social studies course on Canada and Latin America, and this short test attempted to measure their understandings of some of the indefinite terms that they had encountered. Quotes were taken from their text books and the children were asked to interpret them. The first item was this: " 'Many of the plants in Canada use hydro-electric power'. About how many does this mean to you?" The answers ranged from four to ten thousand. Another question was: " 'Large numbers of the people (of Quebec) are French-Canadians'. About how many do you think are French-Canadians?" The answers ranged from 180,000 to 7,000,000, despite the fact that these children had the understanding that there are about 14,000,000 people in Canada. Another question included: "Montreal is an old city founded long ago by the French." The children were directed somewhat in this question with the following multiple choices:

[5] Buswell, G. T. and John, Lenore, *The Vocabulary of Arithmetic*, Supplementary Educational Monographs, No. 38. Chicago: University of Chicago Press, 1931, chapters 3 and 4.

[6] Reported in Horn, Ernest, *Methods of Instruction in the Social Studies*, Chas. Scribners and Sons, New York, 1937.

1. Montreal is about a hundred years old.
2. Montreal is about five hundred years old.
3. Montreal is about three hundred years old.
4. Montreal is about a thousand years old.

Each one of these choices was selected by some children. Four children indicated that Montreal was a thousand years old, which would have put its founding date at 954, somewhat antedating the discovery of America. Perhaps these illustrations point up the fact that quantitative elements in other curricular areas in the elementary school, particularly the social studies, are extremely abundant. And, we might further draw this generalization that many of these concepts, particularly the indefinite ones, are not handled very efficiently by the children.

What Contribution Can Arithmetic Make to Understanding in the Content Areas?

First, it should be emphasized that effective instruction in arithmetic goes beyond the teaching of operations and vocabulary. Indeed, perhaps the thing that ought to be stressed is the ability to think quantitatively. One means of doing this is to establish with the children reference points which will enable them to deal with these ideas. For example, a large farm in Vermont is different from a large farm in Saskatchewan. In addition, a thick coat of ice in Greenland is different from a thick coat of ice in Illinois. In other respects, teachers might deal with something known to the children as being an acre. In fact, a good way of dealing with this is, if your playground is large enough, is to have the children step off an acre. A child can be placed at each corner of the acre. All the children looking at this will undoubtedly get some idea of the size of an acre, and none of them could possibly conceive of an acre as being even remotely the size of the State of Iowa. It is particularly important that the unit to be used as a standard of reference be appropriate. As Spitzer[7] points out, it is of no use to the child to know the distance of one foot in determining the distance of a linear mile even though he may know that a mile contains 5,280 feet. Perhaps the most important thing that can be done in the content areas to deal with these terms is to make the children aware of them and hold them to the meaning that these quantitative ideas are intended to convey. A good deal of this work of sensitizing children to quantitative ideas can be done in the arithmetic time. This will then free the other curricular areas from the annoyances of interruptions, stopping instruction to point out ideas that can be better taught in the arithmetic period, and in general impeding the work in the content areas by dealing with them as such and by keying them in at the appropriate times. For example, something like this can be done:

> "Boys and girls, yesterday when we were discussing the fact that Canada has a population of approximately 14,000,000 people, we had some difficulty in comparing this with something that was a little closer to us. Now, how many people live in the state of Minnesota? How many more people live in Canada than live in Minnesota?"

In this fashion a comparison of the number of people can be made; and then to make this comparison more fruitful and to add some meaning to the social studies, it might be pointed out that the area of Canada is many times the area of Minnesota and yet, despite this immense space, there are relatively few people living there. In dealing with the density of population, this operation can be pointed out in arithmetic classes; it would serve as a good review for long division; it would help teach children this operation so that when they were confronted with these problems in the social studies, they could attack them with confidence and success. Thus, sound instruction in arithmetic, together with a knowledge of what is going on in the content fields, will combine to make more fruitful the teaching of arithmetic by means of supplying reasons for doing certain things in the arithmetic period and by giving children means of working out their problems in the content area. There should be no such thing as arithmetic versus the social studies or arith-

[7] Spitzer, Herbert F., *Teaching of Arithmetic*, Houghton-Mifflin Company, Boston, 1954.

metic versus science; the two are extremely complementary.

What Contributions Can the Content Fields Make to Arithmetic?

Perhaps the foregoing remarks have indicated that the content fields can make a contribution to arithmetic by the very nature of the fact that they have many quantitative elements. However, the method of dealing with these situations in the content areas will influence heavily the enhancement of arithmetic instruction in the general curriculum. Those people who favor incidental teaching have made many claims for this approach, many of which, unfortunately, are not documented. In many studies reporting the effect of incidental learning it was found that teachers did a considerable amount of systematic teaching of arithmetic which resembles very closely those practices which we call "the meaning approach." What this simply means is that without systematic attention to arithmetic, experience units alone cannot run the gamut of the developmental skills needed for arithmetic instruction. Some people have tried to assess the incidental learning of arithmetic by constructing special units. Some of these are loaded with mathematics. In the hands of a good teacher it would be expected that the children's arithmetic would be enhanced. Certainly, when units are highly motivated, are socially significant, and filled with arithmetic, arithmetic is bound to improve. This improvement will be enhanced even above this if there is sound arithmetic instruction going on in an arithmetic period. The point that ought to be raised at this time is simply this: Units in the social studies and units in science, indeed teaching in any other field, ought to be determined on the basis of what is to be learned from these fields. Thus, a social studies unit ought not to be chosen because of the contribution it can make to arithmetic. It should be chosen in terms of the contribution it can make to the social studies. The relationship with arithmetic need not be limited if the selection is made on this basis, for no matter what kinds of units are developed in the social studies and in science, the vast number of quantitative terms indicate that the arithmetic instruc tion will be hard put to keep abreast of the emerging arithmetic concepts in the content fields. There will be no slow down in arithmetic integration. The point is simply that teachers do not need to go out of their way to select units for their arithmetical value because all units, if properly taught, have this value.

EDITOR'S NOTE. The aim of the elementary school is to educate our young people so they can be intelligent in the society into which they are born. This society is not as compartmentalized as the categories reading, writing, arithmetic, and geography would suggest. As Dr. Curtin points out, there are many uses of arithmetic in other school subjects and these uses frequently depend upon ideas, concepts, information, and principles rather than on technical computations. Quantitative terms are sometimes indefinite and have different meaning in different context. For example, *old* as used with other words such as man, city, potato, and cheese takes more specific meaning in association. Thus it would seem only sensible to develop meanings in the many associations that arise in the elementary school. It must be remembered that the pronouncing and spelling of words does not constitute an understanding of the concepts these words should suggest. The role of the teacher in stimulating children to develop ideas is paramount. The teacher must know when it is opportune to explore and extend an idea. She should not in her innocence attempt to teach the process of long division in the first grade just because an experience of the pupils called for this process. In a good modern elementary school the service of a fine teacher is priceless. Such a teacher must have an excellent background of training in several areas plus a fine sensitivity to children and their development.

The Mathematical Training of Prospective Elementary-School Teachers

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Background of the Problem

MANY WHO TEACH ARITHMETIC in the elementary school are under attack. We are told that the arithmetic programs in many of our schools are far from desirable. In fact, in a recent yearbook published by the Committee on the Teaching of Arithmetic of the National Society for the Study of Education, it is stated that in spite of a large number of improvements in materials of instruction and in spite of the research and publication of findings in the psychology of the subject, arithmetic is taught so badly in so many schools that people rarely study any branch of mathematics further than the minimum demanded by requirements.[1]

It has been suggested by some educators that a major factor in the failure of students to master arithmetic is the ineffectiveness of their teacher to present a meaningful program in the subject. Research has led not only to a new concept of arithmetic but also to new ideas as to how it is best learned and taught. Many of these ideas were organized into the "meaning" theory and presented by William A. Brownell in an article in the Tenth Yearbook of the National Council of Teachers of Mathematics in 1935. This theory has now been widely accepted by mathematics-education specialists and it is now fashionable to suggest that elementary-school teachers should teach arithmetic meaningfully. But the agreement in theory does not seem to be matched by a corresponding agreement in practice.

Many teachers have apparently lacked an understanding of the subject necessary to interpret and implement the new theory at the level of practice in the classroom. Professor Brownell, in 1945, pointed out that this might be the case. In discussing the implications of the acceptance of the meaning theory he stated that the teacher who knew arithmetic only as it was taught in the first three decades of this century would be in no position to expound the subject in its meaningful aspects.[2] Dean E. T. McSwain, Northwestern University, has expressed the opinion that modern arithmetic teaching has failed because many teachers have never come to a meaningful understanding of the rules which they have attempted to teach by rote to children. C. V. Newsom, while serving as Associate Commissioner for Higher Education, State of New York, reported that all too frequently teachers in the elementary grades are hardly a jump ahead of their alert students and that many teachers have confided in him that they lack confidence before their classes in approaching various arithmetical concepts.[3] Vincent Glennon, in a summary of a portion of his doctoral study completed at Harvard in 1948, concluded that the elementary-school

[1] D. Banks Wilburn and G. Max Wingo, "In-Service Development of Teachers in Arithmetic," *The Teaching of Arithmetic*, Fiftieth Yearbook of the National Society for the Study of Education, Part II (Chicago: University of Chicago Press, 1951), p. 253.

[2] William A. Brownell, "When Is Arithmetic Meaningful?" *Journal of Educational Research*, XXXVIII (March, 1945), 482.

[3] C. V. Newsom, "Mathematical Background Needed by Teachers of Arithmetic," *The Teaching of Arithmetic*, Fiftieth Yearbook of the National Society for the Study of Education (Chicago: University of Chicago Press, 1951), p. 232.

teachers whom he tested seemed to understand about fifty-five per cent of the understandings that are basic to the computational processes commonly taught in grades one through six.[4] More recently, in 1952, Jacob Orleans, Director of Research, City College of New York, reported on an investigation in which he employed the hypothesis that a major factor in the failure of children to get closer to an understanding of arithmetic is that their teachers are ignorant of the processes and concepts represented by the short cuts which they teach. He decided that "there are apparently few processes, concepts, or relationships in arithmetic that appear to be understood by a large percentage of teachers."[5]

What is needed, apparently, by many teachers in service and all prospective elementary-school teachers, is a new view of arithmetic—an adult view in which the meanings and understandings of the subject are stressed. Since arithmetic is a part of the basic program in the elementary school, it would seem to be a major function of institutions of higher learning who prepare students to teach in the elementary school to provide such training. F. Lynwood Wren, George Peabody College for Teachers, has expressed the idea this way: that each student preparing to teach in the elementary school has a right to expect that the teacher-training program will provide her with the opportunity to develop the scholarship in arithmetic that will enable her to accept her role as an expositor of arithmetical concepts with a feeling of security and assurance.[6]

Committees of national prominence have recommended that training in background mathematics should be provided in curricula for college students preparing to teach in the elementary school. In 1947, in a report in which this need was discussed, the President's Science Research Board pointed out that "since one of the major handicaps to an effective science and mathematics program in the elementary school is the inadequate preparation of teachers, certain specific steps to improve this preparation are necessary."[7] A committee working under the auspices of the National Commission on Teacher Education and Professional Standards proposed in 1948 that a course in general mathematics at the college level should be required of all teachers, regardless of the grade level at which they plan to teach.[8] A point of view akin to this was expressed by the Commission on Post-War Plans of the National Council of Teachers of Mathematics in 1947. In addressing part of its report to students who might be interested in teaching arithmetic in the elementary school, it informed them that the main requirement would be that they *understand* arithmetic.[9]

In spite of the apparent need and demand for college training in background mathematics for prospective elementary-school teachers and teachers in service, many institutions of higher learning are not providing it. In those institutions that do require students enrolled in their elementary education curricula to complete a subject-matter course in mathematics, it seems doubtful that many of them are de-

[4] Vincent Glennon, "A Study in the Needed Redirection in the Preparation of Teachers of Arithmetic," *The Mathematics Teacher*, XLII (December, 1949), p. 395.

[5] Jacob S. Orleans, *The Understanding of Arithmetic Processes and Concepts Possessed by Teachers of Arithmetic*, College of the City of New York, Office of Research and Evaluation, Publication No. 12 (New York: College of the City of New York, 1952), p. 37.

[6] F. Lynwood Wren, "The Professional Preparation of Teachers of Arithmetic," *Arithmetic 1948*, Supplementary Educational Monographs, No. 66 (Chicago: University of Chicago Press, 1948), p. 90.

[7] *Manpower for Research*, Vol. IV, *Science and Public Policy*, The President's Science Research Board (Washington D.C.: Government Printing Office, 1947), p. 99.

[8] Kenneth G. Young, "Science and Mathematics in the General Education of Teachers," *The Education of Teachers—as Viewed by the Profession*, National Commission on Teacher Education and Professional Standards (Washington, D. C.: National Education Association, 1948), p. 150.

[9] "Guidance Report of the Commission on Post-War Plans," Final Report of the Commission on Post-War Plans of the National Council of Teachers of Mathematics, *The Mathematics Teacher*, XL (November, 1947), p. 324.

signed to meet the arithmetical needs of these future teachers. When considering the reasons why grade teachers may lack the understanding of arithmetic necessary to present a meaningful program in the subject, let us not overlook the training programs provided (or not provided) by the institutions which "prepared" them. This writer believes that Glennon was pointing a finger in the right direction when—in the concluding remarks in his report mentioned above—he suggested that educators should consider his findings as they impinge on the problem of the training of teachers of arithmetic.[10] Too, it appears that Orleans was well within the mark when he concluded that institutions training teachers could "directly influence the prospective teacher's knowledge and understanding of arithmetic," but that the results of his investigation seemed to indicate "that our teacher-education programs have not done an adequate job in this respect."[11] The following excerpt from an editorial in *The Elementary School Journal* calls attention to the situation:

> One of the main difficulties has been that no advanced scholarship in arithmetic is available to elementary-school teachers. . . . There has been little opportunity for elementary-school teachers to become competent in arithmetic in the sense that they can become competent in the teaching of science and history.[12]

The present lack of teacher preparation in arithmetic *per se* is alarming. Educators have indicated that research in this area might contribute to an improved teacher-training program in mathematics for prospective elementary-school teachers. The exact ingredient of the "ideal" college training program for these future teachers will probably always be unknown for the perfect program represents goals that are unobtainable. However, there is a definite need for additional research to determine the merits of the various programs which are now being offered in this area. Such research will require much time and all the skill of our profession. Meanwhile, it would appear that those of us who conduct the courses in mathematics for prospective elementary-school teachers could perform a worthwhile service if we were to examine our own courses in the light of the changing needs of our teacher trainees. This the writer has tried to do.

The Problem

Believing that the better preparation of prospective elementary-school teachers in mathematics subject matter is a prerequisite to an improved program in arithmetic in the elementary school, the writer became interested in developing a college course in background mathematics for such future teachers. Since 1952, he has been attempting to develop an initial college course of this type which would provide the student with an opportunity to develop a broader and deeper understanding of the mathematics she might be expected to teach.

In the initial phase of the investigation, the lack of unanimity among institutions training teachers in their approach to common curricular problems was noted. Because of the influence some of these practices would exert on the type of course in background mathematics an institution might provide for its future grade teachers, it was decided that some attempt at evaluating them should be made. A jury consisting of 70 carefully selected mathematics-education specialists—affiliated with 66 institutions of higher learning in 32 states—was used to determine which of several practices currently employed in college training programs in mathematics for prospective elementary-school teachers were favored by them.[13] A partial summary of their replies to a questionnaire follows.

[10] Glennon, *op. cit.*, p. 395.

[11] Orleans, *op. cit.*, p. 45.

[12] "Scholarship in Elementary School Teaching," *Elementary School Journal*, XLVIII (January, 1948), p. 243.

[13] Dr. H. G. Ayre, Western Illinois University, and Dr. E. H. C. Hildebrandt, Northwestern University, assisted the writer by suggesting names of persons for the jury.

Preparation in Mathematics for Admission to Elementary Education Curricula

The practice.—The college entrance requirements in mathematics for students desiring to enroll in the elementary curricula would seem to be a factor that should be considered in determining the kind of initial course in mathematics subject matter that an institution should provide for its prospective elementary-school teachers. Foster Grossnickle found a variety of requirements among the 129 state teachers colleges—all members of the American Association of Colleges for Teacher Education—whose training programs he reported on in 1951.[14] Twenty-four per cent of these colleges had some prerequisite in mathematics for admission to their elementary curricula and less than 6 per cent of them required their prospective students to present credits for at least two years of high-school mathematics before being permitted to enroll in the college's elementary curricula.

The findings.—The selected specialists were asked this question: "How much high-school mathematics do you feel teacher-training institutions should require for entrance of students preparing to teach below grade seven?"[15] Ninety-two per cent of the 70 educators who replied to this inquiry indicated that they favored some requirement of this type. Sixty-eight per cent of them stated that they believed credit for at least two years of high-school mathematics should be required.

The Requirement of a Level of Proficiency in Arithmetic

The practice.—It is known that some institutions of higher learning require all of their prospective elementary-school teachers to meet a certain level of proficiency in arithmetic before permitting them to enroll in the credit course in background mathematics which is required of all of the students in their curriculum. The extent of this practice is unknown, but it seems likely that such a practice would influence the type of required credit course in background mathematics that an institution should provide for its future grade teachers.

The findings.—In evaluating this practice, 66 per cent of the selected specialists indicated they favored requiring a level of proficiency of this type. An additional 6 per cent of the educators did not respond to the inquiry.

The Differentiated Curricula

The practice.—Some institutions of higher learning have but a single training program for all of their pupils who wish to teach below grade nine, while others provide specialized training for different groups of students depending upon the grade level for which they are preparing to teach. In the past few years, several leaders in the field of mathematics education—as well as members of committees of prominence—have spoken in favor of the differentiated curricula. Some of these educators have advocated that a distinction be made in the training programs in mathematics for students who are preparing to teach below grade seven from those who are planning to teach in grades seven and eight. For example, Grossnickle made the following recommendation:

> The course of training should be differentiated so as to prepare teachers for the kindergarten and the first six grades in one group and teachers for Grades VII and VIII in another group.[16]

Although it may be desirable to have the curricula differentiated for the training of teachers, Grossnickle pointed out that this practice has not been the modal pattern of operation. His 1951 report seems to indicate

[14] Foster E. Grossnickle, "The Training of Teachers of Arithmetic," *The Teaching of Arithmetic*, Fiftieth Yearbook of the National Society for the Study of Education, Part II (Chicago: University of Chicago Press, 1951), p. 208.

[15] Throughout the study, by "prospective elementary-school teacher" the writer referred to a student who was preparing to teach below grade seven.

[16] Grossnickle, *op. cit.*, p. 230.

that about two-thirds of the 129 state teachers colleges whose training programs he studied had but a single general curriculum for students desiring to teach at any level below grade nine.[17]

The findings.—The selected specialists were asked if they believed that a different content in background mathematics (not including methodology) should be provided for those planning to teach in grades seven and eight from that provided for those preparing to teach below grade seven. Fifty-seven per cent of these educators indicated they favored such a distinction and 3 per cent gave no response.

Mathematics Subject Matter Requirements in Elementary Curricula

The practice.—Research has revealed that many institutions of higher learning are not requiring a course in mathematics subject matter as a prerequisite to graduation from their elementary curricula. In 1951, W. I. Layton reported on his findings from a study of the catalogs of 85 teacher-training instiitions, which were located in 45 states.[18] He found that but 35 (41 per cent) of these colleges had a requirement of one or more courses in mathematics subject matter for students enrolled in their four year elementary curricula.[19] In the same year, after his nation-wide study, Grossnickle reported these findings:

> Two-thirds of the colleges offering a curriculum for kindergarten and primary grades do not require a course in background mathematics. In more than half of the colleges offering curriculums which prepare teachers for the intermediate and advanced grades, a background course in mathematics is missing.[20]

[17] Grossnickle, *op. cit.*, p. 207.

[18] W. I. Layton, "Mathematical Training Prescribed by Teachers Colleges in the Preparation of Elementary Teachers," *The Mathematics Teacher*, XLVI (December, 1951), p. 553.

[19] It should be mentioned that some teacher-training institutions require a course in mathematics that is a combination of subject matter and methods of teaching arithmetic. Layton found 28 of the 85 colleges requiring courses of this nature.

[20] Grossnickle, *op. cit.*, p. 210.

Layton described this "inadequate preparation" as "alarming." Grossnickle expressed the opinion that it was unfortunate that "too many colleges neither offer a background course in mathematics nor require a minimum standard of competency in arithmetic." He also stated that "this unfavorable situation must be corrected before acceptable minimum professional standards can be established for teachers of arithmetic."[21]

The findings.—Each selected specialist was asked to specify the amount of training in mathematics subject matter (not including methodology) that he believed should be required in a four year curriculum for college students preparing to teach below grade seven. The replies to this inquiry are summarized in Table 1. It reveals that *all of the 65 educators* who responded *indicated that they favored requiring some training in mathematics subject matter for prospective elementary-school teachers.* The median of the amounts of training that they indicated that they believed would be needed to provide students with the *minimum* training in mathematics subject matter needed by prospective elementary-school teachers (exclusive of their training in the methodology of arithmetic) was *3 semester hours.* The median of the amounts of training which they indicated they believed would be necessary for a *desirable* program in mathematics subject matter for such students was *6 semester hours.*

Subject Matter and Professionalized Courses

The practice.—The training in mathematics subject matter provided by institutions of higher learning for prospective elementary-school teachers is generally presented in one of two ways—either alone, in a so-called subject matter or content course, or combined with the methodology of the subject, in what the writer will call a professionalized course. In 1951, Layton re-

[21] Grossnickle, *op. cit.*, p. 220.

TABLE 1*

TRAINING IN MATHEMATICS SUBJECT MATTER RECOMMENDED FOR FUTURE TEACHERS BY THE SELECTED SPECIALISTS

Teaching Objective	Minimum Program			Desirable Program		
	No. of Replies	Median (S.H.)	Range (S.H.)	No. of Replies	Median (S.H.)	Range (S.H.)
Below grade VII.....	65	3	1–10	65	6	2–15

* Adapted from J. J. Stipanowich, "The Development and Appraisal of a Course in Basic Mathematics for Prospective Elementary-School Teachers," Table 7, p. 77. Unpublished Doctoral Dissertation, Northwestern University, 1956.

ported on this practice.[22] He examined 63 courses that were offered to provide prospective elementary-school teachers with training in background mathematics. He found that 35 (56 per cent) of them were subject matter courses and 28 (44 per cent) were professionalized.

The findings.—When asked which of the practices they preferred, the selected specialists seemed to favor the modal practice reported by Layton. Fifty-four per cent of them indicated they favored presenting the mathematics subject matter and methods of teaching it in separate courses. Four per cent stated that they had no preference, and 9 per cent did not express an opinion.

Suggested Modifications in Programs in Mathematics for Prospective Elementary-School Teachers

The practice.—Because it was felt that many of the existing programs were inadequate, the mathematics-education specialists were asked if there were modifications which they believed would improve the "present program in mathematics for students preparing to teach below grade seven" at the institutions with which they were affiliated.

The findings.—The fact that sixty of these seventy educators actually suggested modifications in their own training programs seemed to provide additional evidence that there is a need for improvement in this area of teacher education. Their replies, for the most part, seemed to be concerned with one thing—the lack of training in basic mathematics subject matter. It was found that at 22 of the 62 institutions served by the selected specialists, no course in mathematics subject matter was required in the four year elementary curricula for prospective teachers. As one might expect, then, when these educators were asked to list specific modifications which they believed would lead to an improved program in mathematics education at their own institutions, the need for more training in mathematics subject matter was most frequently mentioned; in fact, it was mentioned 6 times as frequently as any other suggested modification. Other suggested modifications which were frequently mentioned were as follows:

1. Change the content of the subject-matter course so that it more nearly meets the needs of prospective elementary-school teachers.
2. Make a greater effort to provide for the individual differences of prospective elementary-school teachers.
3. Provide better qualified instructors to teach the courses in mathematics for prospective elementary-school teachers.
4. Try to secure a more suitable basic text for use in the subject-matter course for prospective elementary-school teachers.

Content for a Subject-Matter Course for Prospective Elementary-School Teachers

The practice.—There are apparently glaring differences in the content considered in mathematics subject-matter courses provided by institutions of higher education for prospective elementary-school teachers. In

[22] Layton, *op. cit.*, p. 553.

1951, Newsom, in discussing an initial course of this type, expressed this opinion: "The content for a course in background mathematics is a matter of genuine disagreement on the part of even those who are acknowledged authorities in the field."[23] To the writer, it seemed that if certain assumptions were made concerning the nature of an initial course for prospective elementary-school teachers, it might be possible to identify topics that a large per cent of a group of selected mathematics-education specialists would agree should be considered in such a course.

The findings.—In an attempt to verify this hypothesis, the selected specialists were asked to follow these directions in completing Part I of the questionnaire:

> In the part which follows, you are to assume (1) that you are choosing content for a required course in mathematics for college students in the four year elementary curriculum who are planning to teach below grade seven; (2) that the course will not be concerned directly with methods of teaching mathematics since this training is provided for in a separate required course; and (3) that there will be no college or high school mathematics courses listed as prerequisites for this course.

Then, using a detailed check-list as a guide, the selected specialists were asked to do the following:

> Place a check in column A before only those topics you would include in your mathematics course if it were to be a three semester-hour course (about four quarter hours) and were the only mathematics content course required of those students for whom you are developing it.

The topics which were checked in this matter by more than 75 per cent of the 68 selected educators who responded to this inquiry are listed in Table 2. A study of this table reveals that there were 26 topics which at least 90 per cent of the 68 educators who completed this section of the questionnaire indicated they would include in an initial college course of this type. Too, although the check list included items that dealt with algebra and intuitive geometry, the topics checked by more than 75 per cent of the group are arithmetical.[24]

[23] Newsom, *op. cit.*, p. 233.

Recommendations

The information obtained from a study of the replies of the selected specialists is not very reassuring. It provides additional evidence to support the claim that many institutions that train teachers are not providing adequate training in mathematics subject matter for students preparing to teach in the elementary school.[25] But where a large per cent of the carefully selected mathematics education-specialists indicated that they were opposed to what research has reported to be a practice—sometimes the modal practice—of teacher-training institutions, it would seem to suggest lines of attack for improving such training programs. Therefore, in keeping with the opinions expressed by at least a majority of the selected specialists, the following recommendations are made:

1. ADMISSION REQUIREMENTS.—Each student desiring to enroll in an elementary-education curriculum at an institution of higher learning should be required to present at least two units (years) credit in high school mathematics as a prerequisite for admission to the training program.
2. PROVISION FOR A COURSE IN MATHEMATICS SUBJECT MATTER.—Each institution of higher learning that prepares teachers should provide training in background mathematics subject matter for all its prospective elementary-school teachers. This training should be provided for in a separate course from that in which the training in the methodology of arithmetic is presented.
3. REQUIRED TRAINING IN MATHEMATICS SUBJECT MATTER.—Each institution of higher

[24] A majority of the selected specialists indicated they would include topics from algebra and intuitive geometry in the training program in mathematics for prospective elementary-school teachers if time permitted.

[25] A selected specialist, employed at a university in the Mid-West, expressed the hope that this investigation might furnish her "with ammunition to work on the deplorable situation with respect to the training of elementary teachers" at the institution with which she was affiliated. Another, when asked about the training program in mathematics at the teacher-training institution where he was employed, stated that there was no course in background mathematics provided for their prospective elementary-school teachers and added "I am ashamed of this, but at present our hands are tied."

TABLE 2*

TOPICS CHOSEN FOR AN INITIAL COURSE IN BASIC MATHEMATICS FOR PROSPECTIVE ELEMENTARY-SCHOOL TEACHERS BY MORE THAN THREE-FOURTHS OF THE SELECTED SPECIALISTS

Topics	No. in Favor	Per Cent
Arithmetic		
Growth in Number		
One-to-one correspondence	64	94
Counting	66	97
Place value	67	99
Hindu-Arabic Numerals		
The numerals in a decimal system		
Basic meanings	59	87
Positional notation	62	91
Numbers in Our System		
Whole numbers		
The meaning of whole number	68	100
The number scale	63	93
The meaning of zero	67	99
Odd and even numbers	55	81
Common fractions		
Meanings common fractions may express	68	100
Meanings of related terms	63	93
Decimal fractions		
Basic understandings	65	96
Meanings of related terms	65	96
Denominate Numbers and Measurement		
Basic meanings	63	93
Units of measure	63	93
The English system of measurement	56	82
The approximate nature of measurement		
Rounding, accuracy, and precision	61	90
The Fundamental Operations Using Integers		
The theory behind the processes		
Meanings of the processes	62	91
Meanings of related terms	64	94
Important relationships (division as repeated subtraction, etc.)	59	87
The rationale—the "why"	66	97
Checking the results		
Estimating the answer	63	93
Using the inverse operations	61	90
The Fundamental Operations Using Common Fractions		
The theory behind the processes		
Meanings of the processes	64	94
Meanings of related terms (equivalent fractions, etc.)	62	91
The rationale	64	94
The Fundamental Operations Using Decimal Fractions		
The theory behind the processes		
Similarity to operations with common fractions	65	96
Meanings of related terms	65	96
The rationale	63	93
Per Cent		
Its meanings and related terms	67	98
The three cases of per cent	56	82
Aids to Problem Solving in Arithmetic		
Recognizing essential and insufficient data	53	78
Discovering dependencies within the problem	55	81

* Adapted from J. J. Stipanowich, *op. cit.*, Table 10, pp. 85–87.

learning that prepares teachers should require its prospective elementary-school teachers to complete successfully at least a three semester-hour course in mathematics subject matter while in college. As soon as it is practicable, future teachers in this group should be required to complete six semester hours of training of this type.

4. TYPE OF SUBJECT MATTER.—The initial three semester hour course in mathematics subject matter for prospective elementary-school teachers should be primarily an arithmetic course in which the meanings and understandings of the subject are emphasized. Related topics from the fields of algebra and an intuitive geometry should be selected and integrated with the subject of arithmetic.
5. REQUIREMENTS OF A LEVEL OF PROFICIENCY.—Any student enrolled in an elementary-education curriculum who shows that she has not reached a reasonable level of achievement in arithmetic should be required to complete a remedial course before being permitted to enroll for the credit course in background mathematics subject matter required of all of the students in her curriculum.
6. iA DIFFERENTIATED CURRICULA.—The training program in mathematics for teachers should be differentiated so that students desiring to teach below grade seven are trained in one group and students preparing to teach in grades seven and eight are trained in another.

EDITOR'S NOTE. From all reports that have been published it would seem that the requirements in mathematics for elementary school teachers in training are very low. The opportunity to learn a little mathematics is probably better than the minimum requirement. Similarly, the report that less than 6% of training colleges require two years of high school mathematics for admission might be misinterpreted. For example, in one college where no high school mathematics is required for admission only 6% enter with no high school mathematics and 90% have two or more years of such previous training. What is more deplorable is the finding that 22 out of 62 teacher training colleges require no course in mathematics for the future elementary school teachers. Under such circumstances how can one expect a teacher to be more than drill a master of rote learning except for the exceptional cases who learn despite the college? Dr. Stipanowich's consultants are not in harmony with those individuals who say that all one needs to teach anything in the elementary school is a thorough grounding in Child Development nor would they agree that a course in College Algebra is the most fruitful mathematics. They agree that one who teaches arithmetic should know something about this subject and should have a depth of understanding thereof. Is that so revolutionary an agreement? Then why is the situation so deplorable? Who is responsible? How can change be effected? Who will take the lead? Is it in large measure the fault of mathematics teachers themselves? Are mathematics teachers carrying their share of responsibility in curriculum planning for teacher-education colleges? Most thoughtful people will agree with Dr. Stipanowich's conclusions and recommendations. There will be minor differences.

The editor works in a college where six semester hours of mathematics is required for elementary school teaching and where a proficiency examination is used before cadets are permitted to begin practice teaching. Although students may be admitted with no high school mathematics very few come with less than two years and if they are intelligent in mind and attitude, they can be successful. Let us, the mathematics teachers, take the lead in correcting our local situations.

Criteria for Evaluation of Teaching Procedures

ROSS J. SHAW
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I. DO YOUR PROCEDURES, MATERIALS, AND METHODS FACILITATE LEARNING?

New ideas and new gimmicks are worthless unless they contribute to the process of learning. It takes experience to learn to sift the worthwhile ideas from the many new beliefs in teaching methods but the time wasted in trying to teach with some fancy gadget or a new method of procedure that is not basically sound cannot be recalled.

II. WILL THE STUDENTS WORK LONGER AND APPLY THEMSELVES MORE INTENTLY?

Any method that will cause the students to apply themselves with greater effectiveness should be worthwhile. Students learn by studying and the more intent the effort the more effective the teaching should be.

III. WHAT EFFECT DOES THE NEW PROCEDURE HAVE ON THE ACCURACY OF THE STUDENT?

In the teaching of Mathematics accuracy is of extreme importance. Any material, whether concrete, semi-concrete, or abstract is of small value if it adversely affects the accuracy of the students' work. In any competitive game emphasis must be placed on accuracy or the results cannot be satisfactory.

IV. DOES THE NEW PROCESS BRING ABOUT A MORE LASTING RETENTION OF WORTHWHILE MATHEMATICAL FACTS?

Games and gadgets are many times considered as playthings by students and as a result they remember the game but do not remember any of the mathematical concepts that the teacher is trying to develop in the minds of the students.

V. IS THE AMOUNT OF TIME ALWAYS WELL SPENT?

If students are applying twice the amount to your subject as they are to other basic courses then the procedures that you are using are not effective. If mathematics was the only subject taught in school the students would show a high achievement on standard tests but would also show a very poor achievement in other subjects which are basic to their success in school and later life.

VI. DO THE METHODS AND MATERIALS PROVIDE A GOOD UNDERSTANDING OF NUMBER AND THE NUMBER PROCESS?

To have aimless drill, play games, or compete at the blackboard may have some value; however children must understand what they are doing or the process becomes a time consumer drill.

VII. ARE THE NEW PROCEDURES INTERESTING TO THE STUDENTS?

Story number games can be interesting to the first grade student but entirely out of place in the ninth grade. Materials which interest adults can be beyond the interest of a tenth grade student. Too many teachers believe in taking the slow student on his level and keeping him there instead of trying to bring him up to the level of his classmates. There are those students who would do third grade work all year and be satisfied although not interested. To bring the slow or retarded student to achieve a higher level of work requires a great deal of effort and planning. Any procedure used must be interesting and challenging to a slow worker or he will not progress.

VIII. DO YOUR TEACHING METHODS BRING STUDENTS TO A HIGHER LEVEL OF ACHIEVEMENT?

Teaching is not taking the slow student on his level and keeping him there. A fifteen-year-old boy might be satisfied with third grade work but this is not progress. Every effort should be made to help this student to a higher level of learning.

We can keep the bright student playing with concrete objects when he is bored and not interested. He must be challenged or will become careless and a poor student of mathematics. Above average students can work with the abstract intelligently when quite young. Playing with sticks when he should be learning formulas or discovering facts is a deterring influence.

IX. DOES THE PROCEDURE CAUSE THE STUDENT TO USE HIS KNOWLEDGE AT HOME AND AT SCHOOL?

Considerable learning occurs when students explain to parents that which they have learned at school. Using information acquired in the classroom to help in their daily problems makes the work meaningful. School work that isn't meaningful does not bring about worthwhile learning.

X. DOES THE MATERIAL USED CONTRIBUTE TO A GRADUAL EVOLUTION INTO THE NEXT NUMBER PROCESS TO BE LEARNED?

Isolated materials and games are time consumers. Knowledge gained is quickly forgotten unless the material fits into a pattern of progress into new facts and knowledge. Methods and procedures must lead students to discover new facts of mathematics. The discovery method brings about greater retention than the method of presenting isolated rules and facts.

Help Children Discover Fraction Facts

WILLIAM H. GLENN
Pasadena Public Schools, Calif.

IN TEACHING ANY MATHEMATICAL concept, it is always well to begin with experiences which children already have had, in order that their ideas can be freely expressed and certain words can easily become a natural part of their conversation.

In the kindergarten, children can work with actual objects—one-half of a glass of milk, one-half of an apple, one-half of a pie, one-half of a pound of butter, one-half of a foot, and so on. In all cases, they see that two halves make a whole.

Ideas can be extended in succeeding grades by having children bring to the classroom as many things as they can that may be divided in equal parts. They may include such things as an egg carton to show that there are:

6 eggs in a half-dozen,
or 4 eggs in one-third of a dozen,
or 3 eggs in one-fourth of a dozen,
or 2 eggs in one-sixth of a dozen,
or 1 egg in one-twelfth of a dozen.

Milk cartons show that there are:

2 of the $\frac{1}{2}$-pints in a pint,
or 3 of the $\frac{1}{3}$-quarts in a quart,
or 2 pints in a quart (1 pint $= \frac{1}{2}$ quart),
or 4 quarts in a gallon (1 quart $= \frac{1}{4}$ gallon),
or 2 quarts in a half-gallon

The pound of butter shows that there are four quarter-pounds to the pound, and in some cases the paper wrapper of the quarter-pound of butter has printed on it some interesting relationships of the tablespoon and the cup. Relations of this type can be determined:

$\frac{1}{4}$ pound of butter $= \frac{1}{2}$ cup
8 tablespoons of butter $= \frac{1}{4}$ pound

Many relationships with money can also be illustrated, such as:

2 half-dollars = 1 dollar
4 quarters = 1 dollar
10 dimes = 1 dollar

The yardstick is another aid to seeing that 3 feet = 1 yard, and therefore 1 foot = $\frac{1}{3}$ yard.

Definite number experiences with these other objects will reenforce the understandings of the type:

2 halves = 1,
or 3 thirds = 1,
or 4 fourths = 1,
or 2 fourths = 1 half,
or 3 sixths = 1 half,
or 4 eighths = 1 half,
or 2 eighths = 1 fourth.

Children can also be helped to discover fractional relationships if each one has a kit of pie-shaped or rectangular-shaped fractions, each consisting of a set of $\frac{1}{8}$'s, $\frac{1}{4}$'s, $\frac{1}{2}$'s, and 1's. By actually placing these on their desks, each child can match the parts to show that:

2 of the $\frac{1}{2}$'s = 1
4 of the $\frac{1}{4}$'s = 1
8 of the $\frac{1}{8}$'s = 1
2 of the $\frac{1}{4}$'s = $\frac{1}{2}$
4 of the $\frac{1}{8}$'s = $\frac{1}{2}$
2 of the $\frac{1}{8}$'s = $\frac{1}{4}$

These same materials can be used to show how fractions can be added or subtracted in any combination desired. Thus,

$\frac{1}{2}+\frac{1}{2}=1$
or $\frac{1}{2}+\frac{1}{4}=(\frac{1}{4}+\frac{1}{4})+\frac{1}{4}=\frac{3}{4}$
or $\frac{1}{8}+\frac{1}{2}=\frac{1}{8}+(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8})=\frac{5}{8}$
or $\frac{1}{2}-\frac{1}{8}=(\frac{1}{8}+\frac{1}{8}+\frac{1}{8}+\frac{1}{8})-\frac{1}{8}=\frac{3}{8}$

Then fractions should be presented on a linear scale to associate sizes of fractions with

a given unit of length. Such a presentation leads to the development of all four operations with fractions, an introduction to decimal fractions represented on a linear scale, and finally a notion that per cent is based on dividing the unit into 100 equal parts.

Among several of the commercial instructional aids available is The Flan-O-Fraction Kit,* which consists of strips of velour paper that can be used on a flannel board by a teacher to demonstrate the properties of fractions. Ten of the strips are all of equal length, and each represents the same unit length. The side that faces the audience has printed on it the lengths of the fractional parts. The ten strips are shown in the chart on page 252.

As these strips are placed in successive order on the flannel board, the pupils will begin to observe that the following are all equal to one another:

$$1=\frac{2}{2}=\frac{3}{3}=\frac{4}{4}=\frac{5}{5}=\frac{6}{6}=\frac{8}{8}=\frac{10}{10}=1=100\%.$$

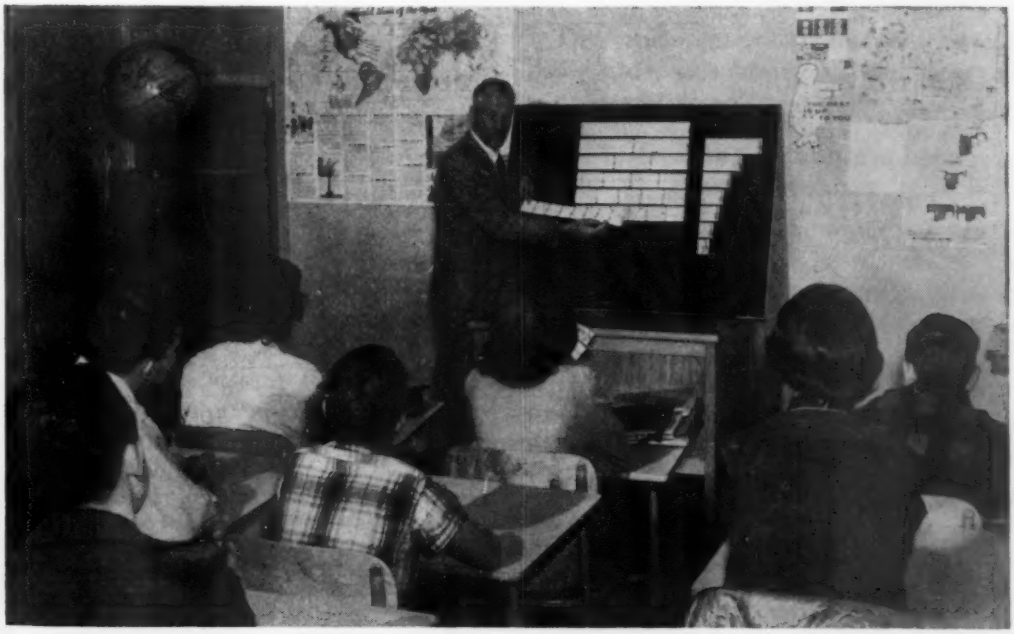
Also there are a number of other values that are similarly related, such as:

$$\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}=\frac{5}{10}=.5=50\%.$$

They will notice, too, that the lengths representing the fractions, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$, become smaller as the denominator gets larger. Doubling the denominator makes the new fraction one-half as large as the original fraction; tripling the denominator makes the new fraction one-third as large as the original fraction; etc.

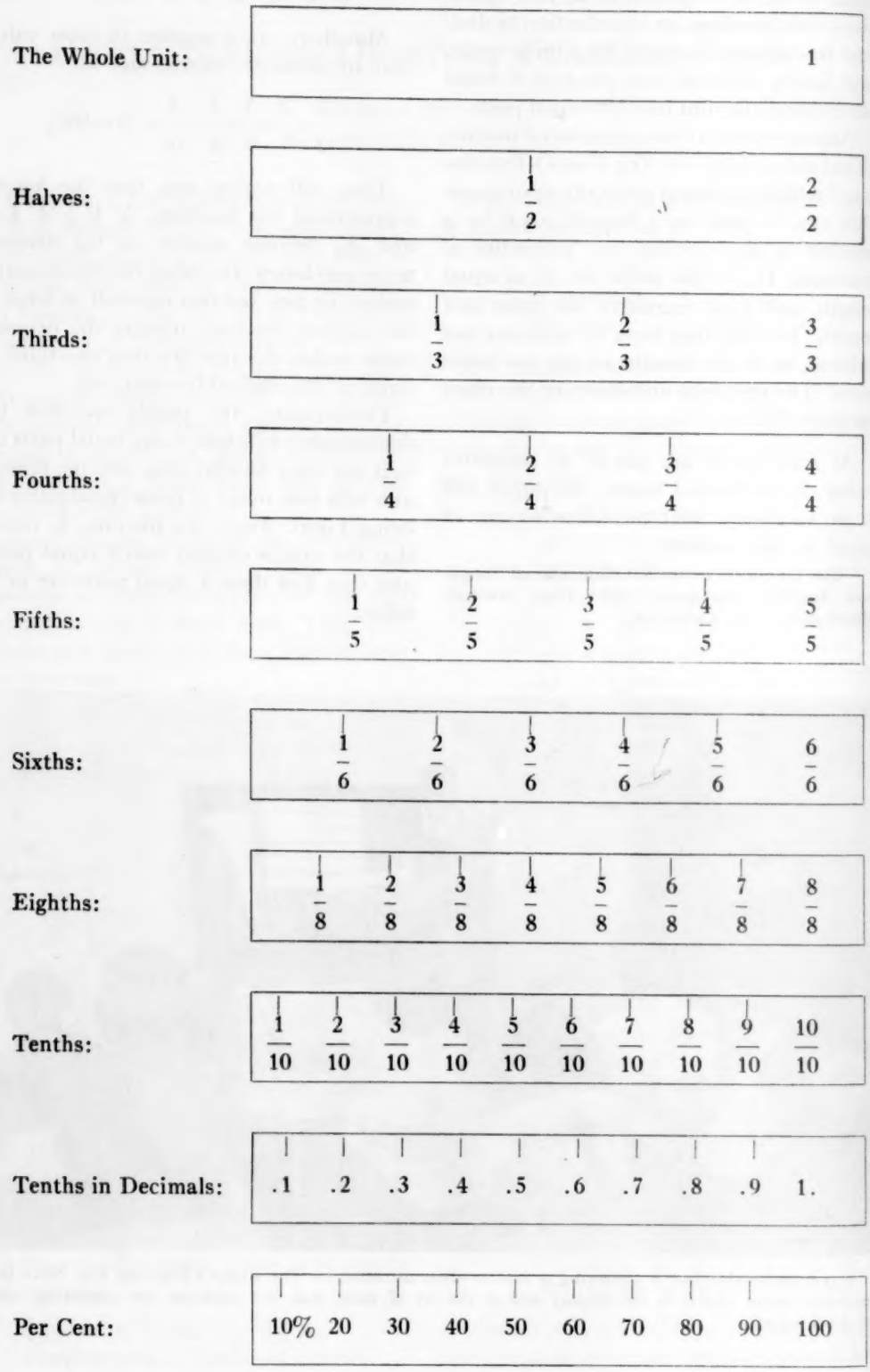
Furthermore, the pupils see that the denominator tells how many equal parts the unit has been divided into, and the numerator tells how many of these equal parts are being taken. Thus, the fraction, $\frac{3}{4}$, means that the unit is divided into 4 equal parts, and that 3 of these 4 equal parts are to be taken.

* The Flan-O-Fraction Kit, Educational Supply and Specialty Company, 2823 Gage Avenue, Huntington Park, California.



A seventh-grade class is observing a teacher-demonstration of The Flan-O-Fraction Kit. Note how successive strips added to the display add to the set of facts that the students are collecting, such as: $\frac{1}{2}=\frac{2}{4}=\frac{3}{6}=\frac{4}{8}$.

Strips Used to Show Relationships



Adding and Subtracting Fractions

One of the next stages of development would be for the pupils to recognize that adding fractions with the same denominator is simply saying how many total parts of a particular size there are. Thus, $\frac{3}{4}+\frac{2}{4}$ means a solution of the type:

3 quarters plus 2 quarters equals (3+2) quarters, which is 5 quarters or:

$$\frac{3}{4}+\frac{2}{4}=\frac{3+2}{4}=\frac{5}{4}.$$

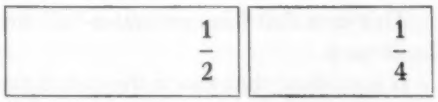
Similarly, in subtraction, $\frac{3}{4}-\frac{2}{4}$ means a solution of the type:

3 quarters minus 2 quarters equals (3−2) quarters, which is 1 quarter or:

$$\frac{3}{4}-\frac{2}{4}=\frac{3-2}{4}=\frac{1}{4}.$$

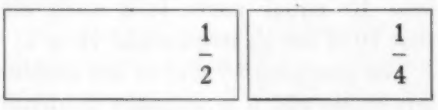
Now suppose that the two fractions had different denominators, such as $\frac{1}{2}=\frac{1}{4}$. For exploring this type of situation, The Flan-O-Fraction Kit contains some additional strips representing each of the fractions, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, and $\frac{1}{10}$.

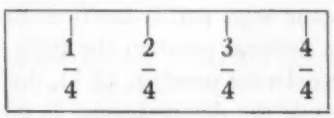
The strip representing the length, $\frac{1}{2}$, is placed on the flannel board; and the $\frac{1}{4}$-strip is placed next to it, in a horizontal line:



The question to be asked of the student is, "Which of the unit strips has a fraction scale that will measure the correct length of these two combined strips?"

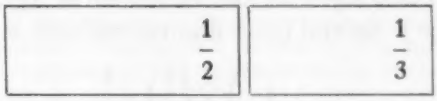
Placing in succession the various unit strips along-side the two, finally leads to the conclusion that the $\frac{1}{4}$ scale measures the sum of the two as $\frac{3}{4}$.

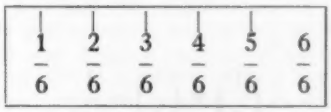




Thus $\frac{1}{2}+\frac{1}{4}$ becomes $\frac{2}{4}+\frac{1}{4}$ which equals $\frac{3}{4}$.

The next most difficult case is: $\frac{1}{2}+\frac{1}{3}$. And when the $\frac{1}{2}$-strip and the $\frac{1}{3}$-strip are placed on the flannel board in a horizontal line next to each other, it is found that the $\frac{1}{6}$-scale gives the answer $\frac{5}{6}$ for the sum.





This can be summarized by writing:

$$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{3+2}{6}=\frac{5}{6}.$$

The corresponding subtraction problem is easily checked with the fraction strips and is shown to be as follows:

$$\frac{1}{2}-\frac{1}{3}=\frac{3}{6}-\frac{2}{6}=\frac{3-2}{6}=\frac{1}{6}.$$

The conclusion is that fractions can be easily added or subtracted if they have the same denominator.

Multiplying Fractions by a Whole Number

Since multiplication by whole numbers is merely repeated addition, the rules for multiplying a fraction by a whole number easily follow. For example: $3\times(\frac{3}{4})$ may be written as

$$\frac{3}{4}+\frac{3}{4}+\frac{3}{4} \text{ (adding } \tfrac{3}{4} \text{ three times)}$$

and by the rules of adding fractions, this becomes

$$\frac{3+3+3}{4} \text{ or } \frac{9}{4}$$

but this is also the same as

$$\frac{(3)(3)}{4} \text{ or } \frac{9}{4}.$$

Multiplying a Fraction by a Fraction

By referring to the strips again, the pupils will observe what happens when a fraction is cut in half. Thus dividing $\frac{1}{4}$ by 2 gives $\frac{1}{8}$.

Another way of approaching this is to say, $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$. This may be written as: $(\frac{1}{2})\times(\frac{1}{4})=\frac{1}{8}$. Several other illustrations such as

$$\left(\frac{1}{2}\right)\times\left(\frac{1}{3}\right)=\frac{1}{6}$$

or

$$\left(\frac{1}{2}\right)\times\left(\frac{1}{5}\right)=\frac{1}{10}$$

will result in understanding that the denominators are to be multiplied.

The students can reach conclusions on how problems can be solved of the type:

$$\left(\frac{3}{2}\right)\times\left(\frac{3}{4}\right)$$

Reference to the strips will show that

$$\frac{1}{2}\text{ of }\frac{3}{4}\text{ is }\frac{3}{8}.$$

Therefore, $\frac{3}{2}$ of $\frac{3}{4}$ is 3 times as large as $\frac{3}{8}$ which establishes that $(\frac{3}{2})\times(\frac{3}{4})=\frac{9}{8}$.

Division of Fractions

The next problem to understand is what is meant when one number is divided by another. If 8 is divided by 4, it can be said that 8 is twice 4. This may be written as: $\frac{8}{4}=2$. We can say that 8 compares to 4 in the same way that 2 compares to 1. This is similar to comparing a half-dollar with a quarter. If the ratio is written,

$$\frac{1\text{ half-dollar}}{1\text{ quarter}},$$

the answer is not forthcoming until the two quantities are expressed in like units. It is necessary to know that 2 quarters may be substituted for 1 half-dollar. Thus:

$$\frac{1\text{ half-dollar}}{1\text{ quarter}}=\frac{2\text{ quarters}}{1\text{ quarter}}=\frac{2}{1}=2.$$

This means that the half-dollar is twice the value of a quarter.

If two fractions are compared by division, such as

$$\frac{\frac{1}{2}}{\frac{1}{4}}$$

the question which is being raised is, "How many times as large is $\frac{1}{2}$ compared to $\frac{1}{4}$?"

The answer is obtained immediately when the two fractions have the same basis for comparison; namely, when they have the same denominator.

$$\frac{\frac{1}{2}}{\frac{1}{4}}\text{ becomes }\frac{\frac{2}{4}}{\frac{1}{4}}=\frac{2}{1}=2.$$

Another example might be to compare $\frac{1}{2}$ and $\frac{1}{3}$ by division:

$$\frac{\frac{1}{2}}{\frac{1}{3}}.$$

The common denominator in this case is 6.

Thus,

$$\frac{\frac{1}{2}}{\frac{1}{3}}=\frac{\frac{3}{6}}{\frac{2}{6}}=\frac{3\text{ sixths}}{2\text{ sixths}}=\frac{3}{2}=1\tfrac{1}{2}.$$

This says that $\frac{1}{2}$ is one-and-a-half times as large as $\frac{1}{3}$.

It is evident that this is the case if the two strips representing $\frac{1}{2}$ and $\frac{1}{3}$ are placed one above the other on the flannel board.

Not until the pupils are thoroughly familiar with obtaining common denominators, should the formal rule of "invert the divisor and multiply" be applied.

Decimal Fractions and Percentage

The $\frac{10}{10}$-strip illustrates the unit divided into 10 equal parts. It is easily checked that 10 of the $\frac{1}{10}$-strips make $\frac{10}{10}$ or 1.

The grouping by tens in the number system shows why it is necessary to identify the unit in some way, and it has been done by placing a decimal point to the right of the units place. In the number, 11.11, the "one" to the left of the decimal point is the unit.

Reading from left to right, each "one" represents a size one-tenth as large as the "one" to its immediate left.

The per cent scale represents the unit scale divided into 100 equal parts so that "one per cent" means "one per hundred" or $\frac{1}{100}$, or 1%.

The complete set of unit scales, from the "1" to the "Per Cent," results in a visual representation of the equivalence of many fractions. For example, students can verify that $\frac{2}{5}=\frac{4}{10}=.4=40\%$ to mention only one.

Conclusion

Understanding the meaning of the numerator and denominator of a fraction is the key to success with working with fractions. What can be done to a fraction without changing its value is extremely important. Objects that have equal parts, such as egg cartons, milk cartons, butter cartons, money, and the yardstick, develop a general awareness of the need of fractions and how relationships exist among certain fractions. Pie-shaped or rectangular-shaped pieces, representing parts of the whole, give still more understandings. The materials in The Flan-O-Fraction Kit or a similar device serve as a constant reminder to pupils that fractions have many, many ways of being written without affecting their value; that operations with fractions sometimes suggest reduction to lowest terms to obtain simpler answers; that in other situations, common denominators must be found before proceeding to take certain steps; that dividing the unit into 10 equal parts gives tenths in the common fraction form and the decimal form; that dividing the unit into 100 equal parts leads to the "per hundred" or "per cent" scale; and that all scales show relationships among each other.

Altogether, visual aids give a mental picture of relationships and help to build the answer to the question, "Why?", by a discovery process.

EDITOR'S NOTE. Mr. Glenn has illustrated very well how we may discover with children basic principles of operation with fractions by using manipulative materials. The particular commercial device he has used is very similar to the fraction strips and charts that many teachers have been using. Some teachers have found it worthwhile to have their pupils participate in the construction of such materials. This gives an added interest and requires an understanding. Certainly, the fraction symbolism we conventionally use has a very particular significance. This significance should be understood as the concepts of fractions are developed. It is worthwhile to recall that symbolism has been created to represent a concept or idea and that this sequence should usually be used in teaching. Later the symbolism of fractions becomes a very useful and powerful tool in computation. As Mr. Glenn has said, the significance of the words *numerator* and *denominator* is a big help in understanding work with common fractions. When it is so easy and so sensible to illustrate the work with fractions one would expect all schools to be using visual-manipulative materials. All that is needed is some strips of heavy paper and a crayon as a start. But one must not limit the development of fractions to a single device. Many teachers prefer "pie fractions."

REVIEW

Arithmetic Games, The Garrard Press, Champaign, Illinois:

Say-it Addition Game	$1.95 ($1.25)*
Say-it Subtraction Game	$1.95 ($1.25)
Say-it Multiplication Game	$1.95 ($1.25)
Say-it Division Game	$1.95 ($1.25)
Make One (Fractions and Percentages)	$1.50 (1.00)

These games, designed by E. W. Dolch, are a possible means of helping children master the basic addition, subtraction, multiplication, and division facts. *Make One* may be used following an understanding of the relationship between common fractions and percentages. Cards are arranged according to difficulty and thus may be selected according to individual needs and abilities.

Since there is a factor of chance in playing these games, it does make the games enjoyable by both the slower student and the better student. These games are another means of providing practice for giving answers to facts more rapidly and more automatic. This, however, is not a substitute for the individual study necessary for each child as a part of his learning program of these facts and the relationships between common fractions and percentage.

* Price in parentheses indicates school price.

Selling Arithmetic to the Pupils

MRS. H. L. LACKEY
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ALTHOUGH QUITE HAPPY with my subject matter assignment (eighth grade arithmetic), it was hard to keep down a feeling of discouragement at the opening of school, upon discovering that the room, in which I should work and meet my classes for the school year, was utterly devoid of the least hint that it was to be, or ever had been, an arithmetic workshop center. Immediately I began to cast about in my mind, seeking ideas for creating an arithmetic atmosphere in this room in which my pupils and I were to spend so many hours during the coming nine months.

In discussing the dreary appearance of the room with my pupils, they soon became enthusiastic in their desire to help "dress up" the room. Pupils as a general rule like to do things. Out of our discussions grew such ideas as making arithmetic posters and booklets, and decorating the bulletin board area above the blackboards with arithmetical figures, signs and symbols.

Such activities called for library research, which in itself proved a fruitful activity, because the pupils discovered for themselves that mathematical thinking and processes had been in use all down the ages. Practically every day some pupil had an interesting idea to contribute to the classes, which he had discovered through research.

Pupils who want to do things see no reason for waiting. So the arithmetic class periods really became activity periods, out of which grew attractive and educational posters and booklets. An attractive, decorative border above the blackboards was made by pupils who cut from heavy, colored cardboard such figures as squares, rectangles, triangles, trapezoids, parallelograms, hexagons, octagons, pentagons, circles, and such arithmetical signs and symbols as plus, minus, multiplication, division, dollar, pi, per cent and equal. One pupil, who had a workshop at home, announced that he was making a surprise; and sure enough—in a few days he came in with the large, six-inch square letters, M. A. T. H. cut from plyboard and painted black, and placed them in the hall above our room door.

In the meantime, the thought kept recurring to me, that in this modern day in the the business world, everyone who has something to sell advertises. I certainly wished to sell arithmetic to my pupils, so, while they were unconsciously selling themselves on it, I decided to use my bulletin boards to furthur the idea. As soon as my pupils saw that I was displaying pictures, graphs, charts, clippings, etc. they, too, began to bring in interesting things for it. The displays were changed from time to time. One display that they enjoyed most consisted of a collection of measuring devices, to which many of them contributed. One seasonal display they enjoyed arranging was that of mathematical Christmas cards.

Displays of pupils' work on the bulletin board is one way of giving recognition for outstanding achievement, either for superior work, neatness, organization, originality or accuracy; and since this was one medium of advertising available to me at all times, I used it frequently. It soon became a daily occurrence for pupils assembling to look for work on display. Expressions on their faces, as they discovered pieces of their own work displayed, were rewarding. They felt it was an honor to have a piece of work so displayed that their classmates could see it.

All the while, I was collecting objects to be used as arithmetic aids, and the pupils assisted with this too. Since a hookboard was not available we used a table for dis-

playing these aids, such as rectangles, prisms (cigar boxes), spheres (balls), rectangular and triangular pyramids, and triangular prisms (made from cardboard), cylinders (mailing tubes), cones, disks (records), cubes (A B C blocks), blackboard compasses and protractor, yard sticks, meter sticks, etc.

From the first day of school my keenest desire was to help my pupils feel and know that arithmetic is important in everything we do. I wanted them to be interested in it—to like it and to be happy while learning it—not just endure it because they must. Some of the pupils, I discovered, were discouraged to begin with—in fact, one or two told me they could not learn arithmetic. Others told me it was very hard for them. So the problem of building up their arithmetic morale was a real challenge. I began trying to help them have confidence in themselves—if they can feel reasonably sure of succeeding in what they attempt, they can learn. In every way possible I showed them I had confidence in their ability to learn arithmetic. At every opportunity I commended those who faltered as well as those who forged ahead.

Gradually splendid attitudes and appreciation were developed. Our classroom became an attractive workshop. The pupils were proud of it because they had played a big part in its creation. They developed more effective learning and study habits, because they were better able to visualize mathematical problems through acquaintance with the many visual aids that we had accumulated. These materials were not used just for display but really became tools for learning. A stimulating atmosphere developed because the pupils developed more confidence and self-reliance through participating in so many activities. Arithmetic became meaningful—something to do—not evade.

EDITOR'S NOTE. Three cheers for Mrs. Lackey whose very enthusiasm for her pupils and for arithmetic is apparent in her writing. She is actually a practical psychologist who understands young people. She sensed a discouraging situation and proceeded to use it as an opportunity to "sell arithmetic." Here is a challenge for all those teachers who are working in schools where pupils are apathetic or may actually hate arithmetic. The role of the inspiring teacher is very very important. It is worthwhile for all of us to cultivate those inner qualities which make our work more enjoyable and fruitful to our students and consequently more rewarding to us as teachers.

The Role of Atmosphere

A visitor to a school soon notes that each classroom has its own distinctive atmosphere. The furniture and decoration of the room are only a part of the climate or educational atmosphere. It is the teacher and the way in which she works with pupils that creates the atmosphere. How does one teacher do this so easily and so well when others experience great difficulty? How have you created a fine friendly school atmosphere where children work and learn and are happy in doing so? What are the several factors? Will you write about this and share your experience with others?

The Use of Graphs for Retarded Children

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GRAPHS ARE VERY EARLY MEDIUMS of a term which in today's education holds a prominent spotlight—VISUAL EDUCATION. Everyone likes pictures as evidenced in the popularity of such magazines as *Life*, *Look*, *Click*, and others. A graph is also a picture. Graphs are pictures measured to exact and comparative scales. They enable adults and children alike to see at a glance facts that it might take several paragraphs of words to describe to the point of full comprehension.

Ever since man first kept records, he has been confronted with the problem of interpretation of numerical data. The graph, which converts into visual form a comparison of various quantities, has met such a need.

Since the purpose of the graph is to present numerical data in visual form, and, since its use has been extended in many fields, the functions of the graph have multiplied. The graph not only compares, it correlates; it analyzes as well as pictures; it forecasts as well as records. It interprets. It shows hidden tendencies which are unforeseen by simply reading a long table of data. We might call it a fortune teller as well as a story teller. Logical conclusions can be drawn from it as well as truthful factualities portrayed by it.

The Mentally Retarded

In planning the curriculum for the mentally retarded , the major consideration is to plan it so that by the time they leave school they will have the type knowledge, the concepts and the experience they will need to help them live as better citizens in their community.[1] Many of the arithmetic skills taught in the advanced grades of the regular school are beyond the comprehension and abilities of the mentally handicapped; much of this material is of little use to them in practical life situations.

In planning the arithmetic program for the mentally retarded,[2] the teacher must integrate number and arithmetic concepts with the social environment and the community life of her pupils. She must motivate learning and knowledge in the fields of social living, science, language arts, citizenship and human relationships. Simple bar graphs are an excellent media for the achievement of these objectives.

In the arithmetic programs for average and superior children, line, bar, circle, and picture graphs are taught before the child graduates from the eighth grade. Since the mentally retarded child generally will achieve between the third and fifth grades in his arithmetic ability, it is advisable to use only the bar graph in the arithmetic program.

Construction of simple bar graphs will fulfill a desire within these children to feel that they are learning and doing the same things in school that the other school children about them are doing.

Graphs tell stories and make pictures. The mentally retarded school child needs every available aid to help him learn. By carefully selecting material within the range of the mental ability of the group, the teacher can make the learning of arithmetic concepts easier and more fun. All school children love to tell stories and draw pictures; the mentally retarded are no different. By having this group participate in the drawing of graphs, they will find school and

[1] Kirk, S., and Johnson, G. O., "Educating the Retarded Child," Houghton Mifflin Co., Boston, 1951.

[2] Applied to those who measure approximately 50 to 75 I.Q.; some authorities include the slow-learning group, 75 to 89 I.Q.

arithmetic a pleasure. The problem of making the retarded child feel that he is not different will be partially solved.

Use Bar Graphs

All children enjoy constructing simple bar graphs, and the construction of them makes interpretation easier. Learning through the use of graphs becomes purposeful and meaningful even to the mentally retarded.

Data for the construction of these simple bar graphs must be chosen from association and recall subjects which will be within the limited comprehension and experience of the group. Such data as:

1) Weights of the pupils in the class
2) Ages of the class members
3) Heights of the pupils of the class
   a) Heights of all members of the class
   b) Heights of the girls of the class
   c) Heghts of the boys of the class
4) Daily grades in some particular subject
5) Classrioom attendance for a week or month
6) Number of birthdays in each month of class members
7) Number of blocks each child walks to school
8) Number of children in each child's family
9) Number of persons in each child's family,

may be used to teach the simplest arithmetic concepts and family relationships. All materials used for the graphs must be already defined by the pupils through daily experiences at home with their families and at play with their peers.

After graphs using this simple data are interpreted and understood, it may be possible to teach science and social development by graphing the following:

1) Daily thermometer readings for a particular hour for a period of several days
2) Cost of simple foods used at home
3) Price of admissions at various entertainments in the community in which the children live.

Some of these may be over-ambitious undertakings with some groups.

The bar graph may be made more appealing if the pupils so skilled draw animals, pictures of people, etc, in the parallel bars across the graph. (Note the graph, next page.) In gathering information for the graph pupils consulted reference books. Then they had to visualize spacing on a page in terms of the numbers in the data. This raised many questions that were discussed and answered. When the graph was completed, it was studied and questions such as the following were asked and answered. "Which pet lives longest?" "Which lives longer, a duck or a chicken?" "How much longer?" "How many times as long does a rabbit live than a hamster?" Discussion led to information which revealed that Joe had a dog 14 years old and this brought out the idea of average and a simple concept of deviation from average. A simple graph not only stimulates interest but also it is a very good medium for developing thinking.

Costello[3] found that the best method of instruction for teaching arithmetic to the mentally retarded was the socialization method, that is, as active experiencing type of activity. Surely the construction of simple bar graphs makes use of this method of learning.

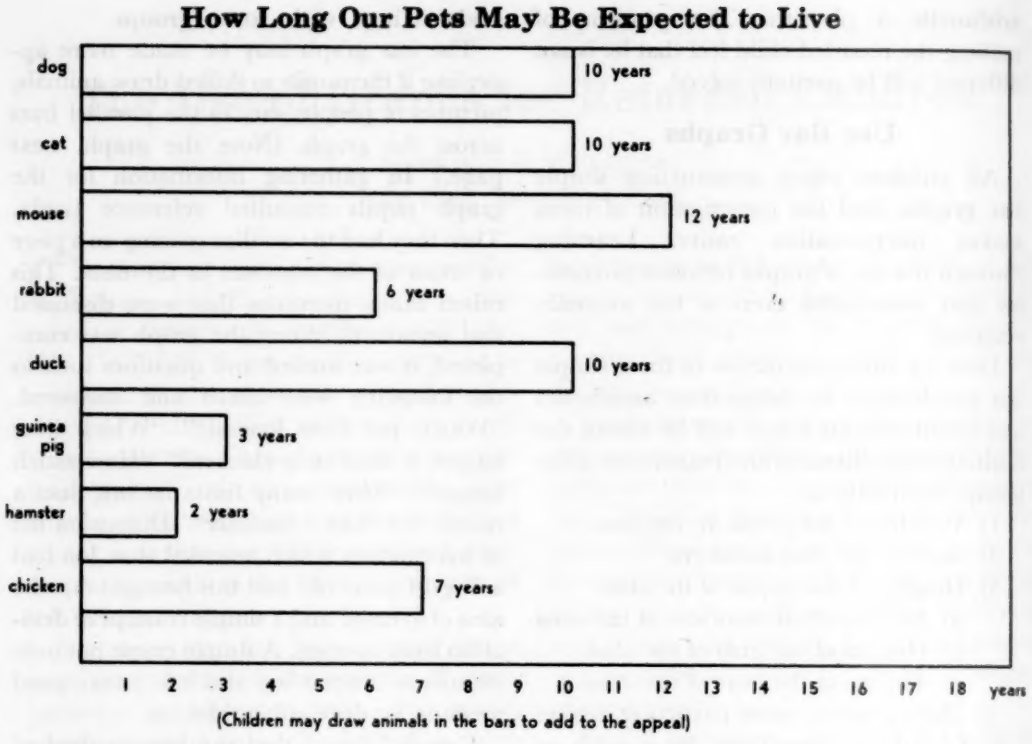
The Education Policies Commission[4] has listed four major objectives of education, namely:

1) Objective of self-realization
2) Objective of human relationship
3) Objective of economic efficiency
4) Objective of civic responsibility.

In general these objectives are applicable to mentally handicapped children as well as to average and superior children. If the instructor chooses the data used in making and interpreting simple bar graphs carefully, all of the objectives of the Education Policies Commission can be accomplished.

[3] Costello, Helen M., "The Response of Mentally Retarded Children to Specialized Learning Experiences in Arithmetic," Doctor's Dissertation, University of Pennsylvania, Philadelphia, 1941.

[4] Education Policies Commission, "Policies for Education in American Democracy," Washington, D. C., National Education Association, 1946.



Enrichment Units in Junior High School Grades

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PROVIDING MORE ADEQUATELY for the rapid learner in arithmetic and mathematics has always been a paramount problem faced by administrators, teachers, and others interested in the education of our youth. Many schools are making excellent provisions for the academically talented in their charge through various enrichment activities, special classes, mathematics clubs, mathematics fairs and contests, and other activities designed to bring about the optimum development of each pupil.

The use of special enrichment units in mathematics and arithmetic for the capable pupil appears to be gaining wide acceptance by schools throughout the nation. Such units are designed for developing greater insight, interest, and motivation in mathematical pursuits. These supplementary enrichment units are used frequently in connection with the regular classroom work but may extend somewhat the scope and difficulty of the material in an attempt to give greater insight into the topic under consideration. For

example, the regular classroom work might be concerned with the fundamentals of our decimal number system. In connection with our base 10 system, supplementary units on other number systems using a base two, five, or perhaps twelve might be studied. Often times, pupils' work with other number systems brings about an interest and motivation which leads to a more thorough exploration of the base ten system or practical applications of mathematical systems used in other phases of mathematics or in electronic computers now in wide use throughout the world.

Can pupils in junior high school mathematics master units of this kind? In Whittier Junior High School in Lincoln, Nebraska, several students became especially interested in an enrichment unit on the binary system. With the use of a self-teaching unit dealing with this base two system and a small amount of individual help from their arithmetic teacher, these capable seventh-grade pupils studied this enrichment unit carefully. As a result, these students not only developed considerable skill in working with binary numbers, but also developed a greater appreciation for the decimal system they had worked with for several years. Another eighth-grade girl in this same school gave a thirty-minute talk to the entire class on the base-eight system. On another occasion an arithmetic teacher presented a special unit on vectors to a seventh-grade boy and asked him to read only the introduction, assuming the main content to be too difficult for her pupil. This talented pupil not only read the introduction, but studied the entire unit, pointing out two errors which had been made in sample problems. Teachers who have used such enrichment units are quick to point out their value and are especially enthusiastic about the new interest and motivations which seems to come about through the use of these challenging units in mathematics.

It appears that special mathematical units for enriching the work for academically talented pupils are gaining considerable favor and will be used widely in the near future. Many curriculum committees, teacher groups, university workshops, and other interested groups are evaluating, inspecting, and developing mathematical units for use in public schools. Some titles of such units now in use in many schools are "The Binary System," "Magic Squares," "Chance," "Mathematical Recreations," "Topology," "Pi," and many others. Teachers and other school personnel will be able to think of many other topics which might well be developed into special units in mathematics valuable to their own teaching situation.

Our schools are faced with the task of developing the mathematical potential of all our pupils to the fullest. In order to accomplish this obligation we must challenge capable pupils; we must develop greater insight, initiative, and interest for them in mathematical pursuits; we must encourage constant growth in mathematics for these pupils by providing opportunities for them to grow; and we must be continually evaluating and designing new materials that might help us more fully attain our primary goal—optimum development in mathematics for all pupils. The units described in this article may give teachers and administrators some idea of a technique that has been successful for many teachers in providing more adequately for the capable pupils in arithmetic and mathematics in their charge.

EDITOR'S NOTE. The editor would like to publish several "enrichment topics." These may be designed for various grade levels and should be outlined briefly and suggest some of the source and reference materials that are useful. The teacher must possess a certain resourcefulness in his stimulation of the pursuit of enrichment. All too many better pupils have been marking time for lack of a challenge and of a little counsel. We are thankful to Mr. Norton for opening this worthy topic. Who will carry the discussion farther?

Why Invert the Divisor?

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CERTAIN OPERATIONS in the fundamental processes seem to present difficulties to students which are often either inadequately removed by the teacher of arithmetic or are left completely unexplained. This would appear to be particularly true in the case of explaining the process used in division of fractions. A survey of the students in my teacher training classes in arithmetic usually shows that the students have had little if any explanation as to why the inverting of the divisor in division of fractions is justified.

Since the process of division of fractions can be considered as being the inverse of the multiplication process, the following approach to the problem of explaining the "why" of inverting the divisor would seem to have merit. Just as a subtraction process may be considered as that of "undoing" an addition which was previously performed, the process of division may be considered as that of "undoing" a multiplication. This concept would seem to be especially desirable in explaining the inverting of the divisor in division of fractions.

Begin with Multiplication

The multiplication process which precedes that of division of fractions must be thoroughly understood both with whole numbers and with fractions. It is best to start with whole numbers to build up the desired background for the reasoning involved. A simple product such as $12=3\times4$ is used for illustrative purposes. This is generalized as:

$$12=3\times4$$

or

$$(\text{a product})=(\text{factor}_1)\times(\text{factor}_2)$$

When we "undo" this multiplication, we obtain

$$\frac{\text{a product}}{\text{factor}_1}=\text{factor}_2 \qquad \frac{12}{4}=3$$

or

$$\frac{\text{a product}}{\text{factor}_2}=\text{factor}_1 \qquad \frac{12}{3}=4.$$

These also may be written as:

$$(\text{a product})\div(\text{factor}_1)=\text{factor}_2 \quad (12\div3=4)$$

or

$$(\text{a product})\div(\text{factor}_2)=\text{factor}_1 \quad (12\div4=3).$$

We thus see that there are *two* division problems associated with this *single* product concept.

$$\frac{12}{3}=4 \text{ is really } \frac{4\times3}{3}=4 \text{ or } 4\times\frac{3}{3}=4\times1=4.$$

We are thus illustrating that reducing the fraction $\frac{12}{3}$ to lowest terms is the process of dividing both numerator and denominator by the factor 3. This has the effect of removing the factor "3" from the product and thus we "undo" the multiplication.

Similarly we would reason

$$\frac{12}{4}=\frac{4\times3}{4}=3=\frac{4}{4}\times3=1\times3=3$$

and the factor "4" has been removed from the product 12 by the division process.

This concept is generalized as

$$\frac{\text{a product}}{\text{factor}_1}=\text{factor}_2$$

or

$$\frac{(\text{factor}_1)\times(\text{factor}_2)}{\text{factor}_1}=\frac{\text{factor}_1}{\text{factor}_1}\times\text{factor}_2$$
$$=1\times\text{factor}_2=\text{factor}_2$$

or

$$(\text{factor}_1)\times(\text{factor}_2)\div(\text{factor}_1)=\text{factor}_2.$$

While the above procedures with whole numbers seem to be easily understood, the necessary procedures are obscured and seem to be quite difficult to isolate and comprehend when fractions are involved.

Transfer to Fractions

The following examples should help to clarify the concept and justify the procedure. The example

$$\frac{3}{4}\times\frac{2}{3}=\frac{1}{2}$$

is especially good for illustrative purposes.

As above

$$\frac{3}{4}\times\frac{2}{3}=\frac{3\times2}{4\times3}=\frac{6}{12}=\frac{6\times1}{6\times2}=\frac{6}{6}\times\frac{1}{2}=1\times\frac{1}{2}=\frac{1}{2}.$$

The product "$\frac{1}{2}$" has been obtained by reducing the indicated product to lowest terms.

The above problem leads to the following division problem:

$$\frac{1}{2}\div\frac{2}{3}=?$$

Complete detail of solution as given in previous examples will not be given in the following examples but would be analogous. Using the reasoning developed with whole numbers, we conclude that since

(a product) $\div$ (factor_1) = factor_2 therefore

$$\frac{1}{2}\div\frac{2}{3}=\frac{3}{4}.$$

This solution presupposes that we have just obtained the product "$\frac{1}{2}$" and hence we know that

$$\frac{1}{2}\div\frac{2}{3}=\frac{3}{4}.$$

or specifically that

$$\left(\frac{3}{4}\times\frac{2}{3}\right)\div\frac{2}{3}=\frac{3}{4}.$$

Similar reasoning solves the associated problem:

$$\frac{1}{2}\div\frac{3}{4}=\frac{2}{3}$$

or

$$\left(\frac{3}{4}\times\frac{2}{3}\right)\div\frac{3}{4}=\frac{2}{3}.$$

We are "undoing" the multiplication in each case above but we knew the factors originally used to obtain the product. That is, we knew that $\frac{3}{4}\times\frac{2}{3}=\frac{1}{2}$ because we had just done the multiplication involved.

Next, we might reason in general that the process of "undoing" the multiplication (i.e., the product) may be considered as that of removing the effect upon the product of one of the factors which make up the product so that we may find out what was the other factor involved.

This is further developed by noting that if a number is multiplied by its reciprocal (defined here as the divisor inverted) that the result is always the number *1*. In our example above

$$\frac{2}{3}\times\frac{3}{2}=1$$

also

$$\frac{3}{4}\times\frac{4}{3}=1.$$

In general any factor multiplied by its reciprocal equals *1*

or

$$\frac{a}{b}\times\frac{b}{a}=1.$$

Finally summarizing the details of our division problem:

$$\frac{1}{2}\div\frac{3}{4}=\frac{2}{3}$$

means that

$$\left(\frac{2}{3}\times\frac{3}{4}\right)\div\frac{3}{4}=\frac{2}{3}\times\left(\frac{3}{4}\times\frac{4}{3}\right)\frac{2}{3}\times1=\frac{2}{3}$$

similarly

$$\left(\frac{2}{3}\times\frac{3}{4}\right)\div\frac{2}{3}$$

$$=\left(\frac{3}{4}\times\frac{2}{3}\right)\div\frac{2}{3}=\frac{3}{4}\times\left(\frac{2}{3}\times\frac{3}{2}\right)$$

$$=\frac{3}{4}\times 1=\frac{3}{4}.$$

Now we must consider that usually we do not know as we did above just what the factors were which make up the product (or multiplication result) which we are expected to "undo" by our division process. However, the process of division involved should be completely analogous and we can proceed as we did above.

In general then, we can remove the effect which a factor has had in the obtaining of a given product (i.e., the dividend) by multiplying the product by the reciprocal of the known factor (i.e., the divisor) to obtain the obscured factor (i.e., the quotient). This is the so-called division process of inverting the divisor and multiplying. This will accomplish the "undoing" of the multiplication.

A further example is probably necessary to clarify this. Consider the example

$$\frac{2}{9}\div\frac{3}{4}=?$$

We must consider $\frac{2}{9}$ as being a fraction reduced to lowest terms. Also we know that $\frac{2}{9}=\frac{3}{4}\times(\text{factor}_2)$ because (a product) = (factor$_1$) × (factor$_2$). We must "undo" this multiplication which resulted in the product "$\frac{2}{9}$" by removing the effect of the factor $\frac{3}{4}$ upon this product. This is done by noting that $\frac{3}{4}\times\frac{4}{3}=1$. We reason that $\frac{8}{9}\times\frac{4}{3}=\text{factor}_2$ since the factor $\frac{3}{4}$ is contained in the product $\frac{8}{9}$.

Finally

$$\frac{2}{9}\div\frac{3}{4}=\frac{2}{9}\times\frac{4}{3}=\frac{8}{27}.$$

We have thus reasoned that factor$_2$ must be $\frac{8}{27}$. This means that

$$\frac{2}{9}=\frac{3}{4}\times\frac{8}{27}$$

or

$$(\text{a product})=(\text{factor}_1)\times(\text{factor}_2).$$

Thus having completed the problem, we can see now the detail which was previously obscured due to our knowing the product originally but only after it had been reduced to lowest terms. That is

$$\frac{2}{9}\div\frac{3}{4}$$

$$=\left(\frac{8}{27}\times\frac{3}{4}\right)\div\frac{3}{4}$$

$$=\frac{8}{27}\times\left(\frac{3}{4}\times\frac{4}{3}\right)=\frac{8}{27}\times 1=\frac{8}{27}.$$

For completeness the following discussion is included:

In general

$$\frac{c}{d}\div\frac{a}{b}=?$$

$$(\text{a product})\div(\text{factor}_1)=\text{factor}_2$$

$$\frac{c}{d}\times\frac{b}{a}=\frac{cb}{da}\ (i.e.,\ \text{factor}_2).$$

This implies however that in general factor$_2$ may be reduced to lowest terms, further "observing" the detail of the problem. We now know that the original multiplication problem resulted in the computation

$$\left(\frac{cb}{da}\times\frac{a}{b}\right)$$

which reduces to the (c/d) which we saw in the problem as given.

Thus:

$$\frac{c}{d}\div\frac{a}{b}=\left(\frac{cb}{da}\times\frac{a}{b}\right)\div\frac{a}{b}=\left(\frac{cb}{da}\times\frac{a}{b}\right)\ \frac{b}{a}$$

$$=\frac{cb}{da}\times\left(\frac{a}{b}\times\frac{b}{a}\right)=\frac{cb}{da}\times 1=\frac{cb}{da}.$$

The *literal* interpretation is perhaps too difficult for grade school students to follow

but is included merely to generalize the process as much as possible. The writer does not feel that at the sixth grade level the above explanation as to why the divisor is inverted in doing the division process with a fraction as a divisor, can be or should be substituted for the usual more or less superficial explanation given.

The concept, when numbers are used, is not too difficult to grasp, however. Basically we are merely saying that division is a process which will undo a multiplication which had been previously performed. This requires that we remove the effect which the known factor (i.e., the divisor) has upon the product. We accomplish this by multiplying the known product (i.e., the dividend) by the reciprocal of the known factor (i.e., the divisor) to obtain the desired, but obscured factor (i.e., the quotient) which is also contained in the product.

EDITOR'S NOTE. Mr. Hannon is concerned with the fact that his students who are high school graduates do not understand the "why" of inverting the divisor. They may have had an explanation of this in the elementary school but did not grasp it and forgot what they did learn. Mr. Hannon's explanation is logical and by analogy. It should appeal to young adults and to better pupils at the sixth and seventh grade levels. It is a good explanation for teachers. It is not one that most pupils in grades five and six would follow but the logic would help pupils to accept the conclusion as reasonable. There are other methods of explaining the inversion of the divisor that are perhaps easier for pupils to comprehend. Division of fractions is one of the topics in which pupils at the lower levels seem to gain only a partial understanding. Many teachers merely show them how to do the process and say that this is something that they will understand better when they are older and have learned more mathematics. At the upper levels we ought to provide an explanation for such topics.

BOOK REVIEW

A Short Dictionary of Mathematics, C. H. McDowell. New York, New York, Philosophical Library, Inc., 1957. Cloth, xiii+40+63 pp., $2.75.

The introduction, "A Short Historical Survey of Elementary Mathematics" is written by Henrietta O. Midonick. This contains some interesting background material for the elementary teacher.

The first 40 pages refer to definitions from arithmetic and algebra. The second part of the book refers to definitions from geometry and trigonometry.

The sketches and examples in the second part add a great deal of interest and information to the little book. The second part should find a place on the reference shelves of teachers of junior high school mathematics.

The American reader will find some differences in word meanings and names because of the British origins of the book.

The discussion of the associative law on page 3 is confusing to the novice for it includes the notion of the commutative law also. I find somewhat simpler illustrations at all points of the discussion of the number laws would be helpful. The statements of irrational decimal approximations are not consistent. At some points the indication of continuation is missing. Page 4 contains an illustration of this. On page 7, i^3 should equal -1 not 1. I find the definition of cross multiplication a bit dangerous for the study of one who is not previously well acquainted with the pitfalls. On page 11 the discussion of divergent series omits consideration of decreasing indefinitely. All points of discussion of exponents and indices would be improved by "taken as a factor" rather than "multiplied by itself." Page 13 has an illustration of this. In discussing division of like bases the order of subtraction of exponents is not indicated. The definition of natural number could be improved. The definition of absolutely convergent series should be cleared up. There are some several misprint errors which will probably be cleared up in another edition.

The brief tables of British measure, the symbol definitions, the Roman numerals and the Greek alphabet are all useful bits of information not often found in such a handy form for personal reference.

DOROTHY SWAN

BOOK REVIEW

Understanding Arithmetic (revised edition), Robert L. Swain. Rinehart and Company, Inc., 1957. Cloth, xxi+264 pp., $4.75.

This is a professional book dealing primarily with the content of arithmetic rather than with instructional method. It is intended for use chiefly in the pre-service preparation of elementary school and junior high school teachers but may be used to advantage with certain other student groups as well.

This book reveals numerous variations from more or less similar volumes in terms of the scope of content covered and its organization and treatment. Without doubt the most significant difference in this connection is the extent to which Swain interprets numerous aspects of arithmetic in terms of ideas from "modern mathematics," particularly the concept of *set*. This will be the most influential single factor in determining a reader's reaction to *Understanding Arithmetic*.

The notion of a set is introduced relatively early in the book (Chapter 2) "Sets and Numbers." Some of the unique symbolism or notation associated with sets and subsets is used to advantage. *Number* is more or less re-defined in terms of the set concept, following an earlier distinction between *number* and *numeral*.

Swain admittedly does not hold consistently to the essential difference between the terms number and numeral in the way in which he uses them: "This important distinction between 'number' and 'numeral' sometimes escapes notice, because the term 'number' is widely used to refer to either concept. In this book we shall make the distinction whenever it seems wise to do so, but at other times we shall follow the common practice" (page 4).

Subsequent sections of the book are developed from set concepts to varying degrees, with greatest use of these ideas being made in Chapter 3 ("Number Operations"). Here the principal operations of addition and multiplication, and the inverse operations of subtraction and division, are treated from a *set* viewpoint.

In dealing with the union of two disjoint sets, X and Y (e.g.) Swain relies exclusively upon the symbol "$X+Y$" and does not introduce and use the more unique symbol "$X \cup Y$" to denote this idea of union. This is but one of numerous instances in which Swain admittedly deals with certain aspects of set theory without being highly rigorous, either conceptually or notationally or both. In some instances this lack of rigor is acknowledged explicitly. In other instances there is no explicit acknowledgment of a lack of rigor that is quite intentional on the author's part and that is justified in his eyes in light of the mathematical background and needs of the main audience he is addressing.

In the early work with sets and their application to number operations, set concepts also are used as the basis for developing the commutative, associative, and distributive laws. Furthermore, set concepts form the basis for distinguishing between two interpretations of the 0 symbol. In reference to the system of whole numbers (designated by the numerals 0, 1, 2, 3, 4, $\cdots$), 0 is interpreted in relation to the idea of an *empty* set. Then in reference to the system of integers (designated by the symbols $\cdots$, -3, -2, -1, 0, 1, 2, 3, $\cdots$), 0 is interpreted in relation to the idea of a *null* set.

Swain's subsequent treatment of computational procedures for performing the fundamental operations within the framework of our numeral system (Chapters 4 and 5) is disappointing to this reviewer. For one thing the chapters omit many logical sequential steps in the development of ultimate computational procedures and thus take too much for granted in relation to the student's own mathematical background. For another thing the author's discussion of methods of

compound subtraction is both inadequate and misleading. This is particularly true of his interpretation of the vast body of research on the relative merits of the *decomposition* and *equal additions* procedures—which Swain elects to identify by expressions other than these two which are so commonly accepted and used in our professional literature. His subsequent discussion of methods of estimating quotient digits when dividing by two-digit divisors is much more satisfactory. Here again, however, he elects to use expressions (*one-step rule* and *two-step rule*) different from those commonly accepted and used in the professional literature (*one-rule method* and *two-rule method*, or *apparent method* and *increase-by-one method*).

Swain's handling of computational procedures in his chapters on "Fractions" (Chapter 8) and "Decimals" (Chapter 9) is as disappointing to the present reviewer as was his treatment of such techniques in Chapters 4 and 5, mentioned previously. The same major reason applies: many logical sequential steps in the development of ultimate computational procedures are omitted, thus taking too much for granted in relation to the student's own mathematical background. Also, in Chapter 9, the discussion of techniques for placing a decimal point in a quotient is considered inadequate.

On the other hand, this reviewer is impressed favorably with numerous aspects of Swain's material dealing with duodecimal and binary scales of notation (Chapter 6), with the theory of natural numbers (Chapter 7), and with work relating to measures and approximate data (Chapters 11 and 12). His inclusion of selected theorems, etc. and their proofs is a particularly strong feature.

In summary, we may expect mixed reactions to *Understanding Arithmetic*, particularly from the standpoint of Swain's relative emphasis upon the concept of set. At the one extreme there will be persons who feel that none of this "modern stuff" should have been included. At the other extreme there undoubtedly will be persons who feel that Swain has been much too superficial in his treatment of ideas from "modern mathematics," that he has compromised entirely too much with mathematical rigor.

As far as this reviewer is concerned, either extreme position would be unfortunate and unwarranted. The most reasonable opinion would seem to lie somewhere between these extremes. In relation to the readers for whom *Understanding Arithmetic* is intended, Swain's handling of the set concept and related ideas from "modern mathematics" should be more helpful than not. His efforts in this connection should be welcome, and are deserving of commendation rather than condemnation.

Some things have been mentioned as shortcomings in the eyes of the present reviewer. To other persons, Swain's treatment of the topics in question would be adequate for his intended purposes. Such a difference in reaction would be quite understandable, since for the most part these are matters of opinion rather than fact.

In any event, *Understanding Arithmetic* should represent a worthwhile addition to our professional literature for persons seeking a better understanding of arithmetic content *per se*.

J. FRED WEAVER

Give a Subscription

to

The Arithmetic Teacher

to

a friend or a school

Why Teach Bank Discount?

ALAN RIEDESEL
State Teachers College, Plattsburgh, N. Y.

ONE OF THE AREAS in today's elementary school curriculum that has undergone revision in the past twenty years is the arithmetic program. While there has been much progress much work remains to be done if we are to make the experiences offered of value to the child now and in his adult life.

One area that appears to be of questionable value is the teaching of bank discount in the eighth grade arithmetic program. In this study, eighth grade students learn to compute the interest on bank loans and deduct the amount from the principal to arrive at the amount which the borrower actually receives.

A study of current arithmetic textbooks, those published following 1950, shows that eight out of nine of the more widely used books have from one to four pages of work concerned with the discounting of bank loans. (See Table 1.)

TABLE 1

Textbook	No. of Pages
A	2
B	2
C	3
D	1
E	0
F	4
G	1
H	1
I	2
J	

When present banking procedures are checked, one finds that the majority of bankers make use of tables computing the discount on loans. If the banker in his daily dealings with this topic makes use of tables, it seems that we are misusing the time of the eighth grade student. To include a phase of arithmetic which has little or no value to the student in his present living seems unwise, and will be of questionable value to him as an adult. While this phase of the eighth grade program represents only one to three days work, it still represents time that could be used in experiences of greater value to the student.

Much of the present eighth grade program appears to be a holdover from the time when the eighth grade was the terminal point in education. This is unrealistic today when over 70 per cent of all children enter high school, and 50 per cent finish tenth grade. If we are to keep interest in arithmetic at a high level, it will be necessary to give junior high students meaningful and valuable arithmetic experiences. This does not mean that every phase of the program must be taken from the students' day-to-day experiences, but that material taught should deal with problems that arise during their eighth grade year and the years that follow.

EDITOR'S NOTE. Mr. Riedesel argues that we are misusing the time of pupils when they are finding discounts by calculation when most commercial houses do this by using a table. It is not the computation that is of primary importance, rather, it is the principles and the information that are of great importance to a functioning citizen. Computation is not only a step in the topic, it is also an aid in appreciation and understanding of the procedures. How much bank discount should be taught and the limits of detail and difficulty of the exercises are matters of differing opinion. One can argue that this topic more properly belongs in the senior high school at a level which is several years closer to adulthood. Perhaps the pupil's time in grade eight might be spent more worthily on some other topic. But let us remember that money and the various ways in which money and credit are used in this country are of great importance to practically all of us.

The National Council of Teachers of Mathematics

Report of the Membership Committee

MARY C. ROGERS, Chairman, Membership Committee
Roosevelt Junior High School, Westfield, New Jersey

The Membership Committee of the National Council of Teachers of Mathematics is happy to announce that an all-time high in membership total has been reached, and wishes to thank you for your fine cooperation and support. It has been the assistance of each of you as individuals which has been largely responsible for this accomplishment. The Committee is indeed grateful to you for your helpfulness in this service to the Council.

May we submit to you at this time:

1. A brief report of membership action taken at the Thirty-fifth Annual Meeting in Philadelphia.
2. The latest membership analysis, based on the May 1, 1957 membership count by states received from the Washington Office.

Convention Reactions

The announcement of a 15,175 membership total—based on the February 1, 1957 official count—was received with a great deal of enthusiasm by the Delegate Assembly; by the State Representatives; and by the many other persons in attendance at this —the largest annual meeting in the history of the Council.

Dr. Howard F. Fehr and the Board of Directors of the Council were particularly pleased with what they called the "phenomenal" growth over the past three years. They instructed the Committee to continue the work, using its own best judgment as to immediate goals and procedures. They expressed their sincere appreciation not only for the work of the Committee, but also for the outstanding support and diligent assistance of *all persons* concerned with increased membership. This includes not only the State Representatives and officers of Affiliated Groups, but it also includes each of you primarily whose direct contact with prospective members has been so instrumental in bringing about membership growth.

At the Delegate Assembly, Dr. Fehr expressed his thanks and that of the National Council Board to all persons responsible for the current strong and steady membership growth. He offered his opinion, however, that we had "only scratched the surface" of potential membership strength, and challenged the Group to an ultimate *25,000 membership total.*

In her report to the Assembly, the Membership Chairman called for reactions to this Challenge:

1. Should present goals be maintained with continued efforts to increase membership?
2. Should the 25,000 membership goal be accepted with State Goals raised accordingly? If so, what time limit should be agreed upon in which to reach new goals?

The Assembly voted to accept the 25,000 challenge at once; to support whatever plan for reaching this goal should be recommended by the Membership Committee.

The Membership Committee, in conference with the State Representatives, arrived at a recommendation of a 5-year time limit. This recommendation, approved by the NCTM Board, received generous publicity throughout the Convention. This action sets up the following goals for the next five years:

April 1957–April 1958	17,000
April 1958–April 1959	19,000
April 1959–April 1960	21,000
April 1960–April 1961	23,000
April 1961–April 1962	25,000

Record of Membership Growth

You will be most gratified to note the present high total of 16,181, based on the May 1, 1957 official count; also that this total is 95% of the suggested April 1, 1958 goal of 17,000. In preparing the accompanying membership analysis, we have reviewed with you former State goals, based on the 15,000 total; we have also indicated new State goals based on the 17,000 total to be reached in April 1958. We are reporting specific achievements based on both of these goals. A study of the data makes it quite evident that we were ready to "raise our sights" and that our new goals are very reasonable. With your continued support and assistance, they should be readily reached.

Based on the 15,000 goal, 62% of all states and territories have reached their goals or gone beyond them. These states are:

1. Arizona	275%	17. Connecticut	114%
2. Oregon	172%	18. Maine	113%
3. Nevada	164%	19. Montana	113%
4. South Dakota	156%	20. Wisconsin	112%
5. Canada	148%	21. Mississippi	111%
6. Utah	133%	22. Foreign	109%
7. California	132%	23. District of Columbia	107%
8. New Hampshire	126%	24. Michigan	107%
9. Florida	121%	25. New Jersey	107%
10. U. S. Possessions	120%	26. Maryland	106%
11. Pennsylvania	119%	27. Indiana	103%
12. Washington	118%	28. Oklahoma	103%
13. Idaho	117%	29. Louisiana	101%
14. New York	117%	30. Ohio	101%
15. Colorado	116%	31. Iowa	100%
16. Texas	116%	32. Massachusetts	100%

Based on this same 15,000 goal, membership achievement in these 12 additional states shows 90%–99% of established goals:

1. Georgia	99%	5. Illinois	96%
2. Virginia	99%	6. Kansas	96%
3. Delaware	97%	7. Missouri	96%
4. New Mexico	97%	8. Minnesota	94%
		9. Kentucky	93%
		10. Vermont	93%
		11. Wyoming	92%
		12. Arkansas	91%

Once again based on the 15,000 goal, membership in the following three states shows 85%–89% achievement:

Tennessee	87%
Alabama	86%
Nebraska	86%

Membership in five states has remained relatively low. Apparently in these states there is need for more stimulation of interest, more encouragement and more help from the Membership Committee. Suggestions from present members in these states as to better procedures and more effective assistance from the Committee will be gratefully received.

It is pleasing to see that—based on the new 17,000 goal—19 states and territories have already reached their new goals or gone beyond them. An additional 9 states have achieved from 90%–99% of their new goals. 11 states show 85%–89% of achievement. These states are listed here:

STATES AND TERRITORIES HAVING REACHED THEIR GOALS OR GONE BEYOND THEM

(Based on 1957–1958 Goals)

1. Arizona	243%	11. Idaho	105%
2. Nevada	153%	12. Pennsylvania	105%
3. Oregon	152%	13. Washington	105%
4. South Dakota	137%	14. New York	103%
5. Canada	130%	15. Colorado	102%
6. Utah	119%	16. Texas	102%
7. California	117%	17. Connecticut	100%
8. New Hampshire	112%	18. Maine	100%
9. Hawaii & U. S. Poss.	107%	19. Montana	100%
10. Florida	106%		

MEMBERSHIP ACHIEVEMENT 90%–99%

(Based on 1957–1958 Goals)

1. Mississippi	99%	6. New Jersey	94%
2. Wisconsin	99%	7. Maryland	93%
3. Foreign	96%	8. Oklahoma	92%
4. District of Columbia	95%	9. Indiana	91%
5. Michigan	94%		

MEMBERSHIP ACHIEVEMENT 85%–89%

(Based on 1957–1958 Goals)

1. Iowa	89%	7. Delaware	86%
2. Louisiana	89%	8. New Mexico	86%
3. Massachusetts	89%	9. Illinois	85%
4. Ohio	89%	10. Kansas	85%
5. Georgia	88%	11. Missouri	85%
6. Virginia	87%		

Other indications of progress include:

LEADERS IN MEMBERSHIP TOTALS INCLUDING SUBSCRIPTIONS

1. New York	1,378	9. Indiana	557
2. California	1,163	10. Wisconsin	470
3. Illinois	1,143	11. Massachusetts	442
4. Pennsylvania	1,093	12. Florida	423
5. Texas	778	13. Foreign	368
6. Ohio	747	14. Minnesota	368
7. Michigan	640	15. Virginia	352
8. New Jersey	598		

LEADERS IN MEMBERSHIP TOTALS—NOT INCLUDING SUBSCRIPTIONS

1. Illinois	954	9. New Jersey	433
2. New York	947	10. Wisconsin	356
3. Pennsylvania	817	11. Florida	330
4. California	652	12. Massachusetts	330
5. Ohio	605	13. Virginia	272
6. Texas	527	14. Minnesota	254
7. Indiana	491	15. Kansas	248
8. Michigan	452		

LEADERS IN MEMBERSHIP GROWTH—INCLUDING SUBSCRIPTIONS

(Since May 1956)

1. California	275	9. Indiana	79
2. Pennsylvania	249	10. Florida	77
3. New York	226	11. Illinois	68
4. Ohio	134	12. Canada	64
5. Foreign	127	13. Arizona	59
6. Texas	105	14. Minnesota	57
7. Michigan	96	15. Oregon	54
8. New Jersey	87		

Membership Growth—Members and Subscribers—May 1956–May 1957

	May 1956	May 1957		Goals and % of Goals Reached			
		Individuals	Totals	'54–'57	%	'57–'58	%
1. Alabama	135	107	147	170	86%	192	77%
2. Arizona	106	125	165	40	275%	68	243%
3. Arkansas	106	126	156	171	91%	194	80%
4. California	888	652	1,163	879	132%	996	117%
5. Colorado	168	159	208	180	116%	204	102%
6. Connecticut	220	184	259	228	114%	258	100%
7. Delaware	66	56	69	71	97%	80	86%
8. Dist. of Columbia	214	175	198	185	107%	209	95%
9. Florida	346	330	423	351	121%	398	106%
10. Georgia	152	133	185	186	99%	211	88%
11. Idaho	12	12	21	18	117%	20	105%
12. Illinois	1,075	954	1,143	1,188	96%	1,346	85%
13. Indiana	478	491	557	542	103%	614	91%
14. Iowa	270	215	297	296	100%	335	89%
15. Kansas	285	248	299	311	96%	352	85%
16. Kentucky	98	87	115	123	93%	139	83%
17. Louisiana	236	191	279	276	101%	313	89%
18. Maine	67	63	78	69	113%	78	100%
19. Maryland	269	245	301	285	106%	323	93%
20. Massachusetts	389	330	442	440	100%	498	89%
21. Michigan	544	452	640	600	107%	680	94%
22. Minnesota	311	254	368	391	94%	454	81%
23. Mississippi	106	102	137	123	111%	139	99%
24. Missouri	278	236	316	330	96%	374	85%
25. Montana	59	48	68	60	113%	68	100%
26. Nebraska	126	109	142	165	86%	187	76%
27. Nevada	23	14	23	14	164%	15	153%
28. New Hampshire	74	64	82	65	126%	73	112%
29. New Jersey	511	433	598	561	107%	636	94%
30. New Mexico	79	58	86	89	97%	100	86%
31. New York	1,152	947	1,378	1,181	117%	1,338	103%
32. North Carolina	198	161	209	255	82%	289	72%
33. North Dakota	28	22	35	44	80%	49	72%
34. Ohio	613	605	747	740	101%	838	89%
35. Oklahoma	212	183	245	237	103%	265	92%
36. Oregon	176	151	230	134	172%	151	152%
37. Pennsylvania	844	817	1,093	920	119%	1,042	105%
38. Rhode Island	52	34	55	71	77%	80	69%
39. South Carolina	102	51	105	137	77%	155	68%
40. South Dakota	52	31	56	36	156%	41	137%
41. Tennessee	207	161	213	246	87%	279	76%
42. Texas	673	527	778	672	116%	762	102%
43. Utah	60	39	64	48	133%	54	119%
44. Vermont	41	31	38	41	93%	46	83%
45. Virginia	310	272	352	357	99%	405	87%
46. Washington	226	161	270	228	118%	258	105%
47. West Virginia	77	73	91	158	58%	179	51%
48. Wisconsin	465	356	470	420	112%	476	99%
49. Wyoming	36	26	33	36	92%	41	80%
Totals	13,214	11,301	15,427	14,388	107%	16,305	93%
Hawaii & U. S. Poss.	139	60	89	74	120%	83	107%
Canada	233	175	297	201	148%	228	130%
Foreign	241	145	368	339	109%	384	96%
Grand Totals	13,827	11,681	16,181	15,002	108%	17,000	95%

GREATEST RELATIVE GROWTH

(*Since May 1956*)

1. Idaho
2. Arizona
3. Foreign
4. Arkansas
5. California
6. Oregon
7. Pennsylvania
8. Mississippi
9. Canada
10. North Dakota
11. Colorado
12. Florida
13. Georgia
14. Ohio
15. Washington

STATES WITH CONTINUOUS GROWTH

(*Since May 1956*)

1. Arkansas
2. California
3. Connecticut
4. Illinois
5. Kentucky
6. Mississippi
7. Nebraska
8. Ohio
9. Oklahoma
10. Oregon
11. Pennsylvania
12. Utah

Future Plans

We believe this record of accomplishment is one of which each of us should be justifiably proud. We are confident that the future holds achievements equally commendable; that with your continued support and assistance challenging goals will be reached.

We suggest that future procedures should continue to follow closely those which have been proved most effective in the past.

1. We have found the "EACH ONE WIN ONE" technique to be our most valuable aid. We urge a continuance of its use by *all present members of the Council*.
2. We cannot emphasize too strongly the importance of the *National Council publications* to every teacher of mathematics throughout the country. Currently invaluable is the Twenty-third Yearbook—*Insights into Modern Mathematics*. Let us call attention to the fact that *special prices are made to NCTM members on all Council Yearbooks*.
3. We should call attention to the *National Council membership directory* and the professional prestige afforded through inclusion in this listing.
4. *Prompt renewals* greatly facilitate the keeping of records in the Washington Office and the preparation of reports.
5. You who are *members of the mathematics staff in colleges of education* and similar education centers; you who are *supervisors and/or mathematics department chairmen*; you who are *State Representatives* or *Presidents of Affiliated Groups* are performing an outstanding service to the Council in matters of membership. A continuance and expansion of your fine work will be most sincerely appreciated.

At the beginning of a new year, please accept our sincere good wishes for your greatest professional success at all times.

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THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

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BEN A. SUELTZ, *Editor*

State University Teachers College, Cortland, New York

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Clifford Brewster Upton

May 10, 1877–September 25, 1957

Professor Upton is known to the younger generation of teachers as an author of textbooks in arithmetic, first the Strayer-Upton series and currently the Upton-Fuller. They know him through his contributions to the literature of the teaching of mathematics, respecting his scholarship, his clear exposition and the artistry of his work, and regretting that these monographs are so few. His students—there were many of them in the thirty odd years that he was at Teachers College, Columbia University—are indebted to him for inspiration, guidance, counsel, occasional admonition, and for the standards of performance in study, teaching, and writing that he required.

Mr. Upton spent the first twenty-five years of his life in Michigan, graduating from the Michigan State Normal College in Ypsilanti in 1898, teaching in a high school and at Ypsilanti and earning his bachelor's degree from the University of Michigan in 1902. At Ypsilanti, he was a student of Dr. David Eugene Smith with whom he was to be associated for many years.

In 1902, Mr. Upton became head of the mathematics department of the Horace Mann School, the demonstration school of Teachers College. (Professor Smith had joined the T.C. faculty in the previous year.)

In 1907, Mr. Upton was appointed to the mathematics department at Teachers College, but he retained his close contact with the Horace Mann School and on more than one occasion used its classes for research and study. After periods of study at Göttingen and at the Sorbonne, Professor Upton served Teachers College in dual capacity—teaching in the mathematics department and acting as secretary of the College from 1911 to 1919 and as Provost from 1919 to 1924. He retired from teaching in 1942 and spent the next three years as chairman of the board of the American Book Company.

His published articles include: "Standardized Tests in Mathematics for Secondary Schools" in the report of the National Committee on Mathematical Requirements, 1923; *Studies in the Teaching of Arithmetic*, 1927; "The Use of the Indirect Proof in Geometry and in Life," *Fifth Yearbook of the National Council of Teachers of Mathematics*, 1930; "Making Long Division Automatic," *Tenth Yearbook NCTM*, 1935. The first of the *Arithmetics* with which he was associated was published in 1928.

The holders of the Clifford Brewster Upton Fellowship which the American Book Company established in 1956 are heirs to a high standard in scholarship and in teaching.

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